TWO-DIMENSIONAL BEHAVIOR OF A THIN WEB ON A ROLLER

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Abstract

A web on a roller is usually modeled as a one-dimensional belt in a state of pure circumferential stress. However, most of the important problems in lateral web behavior involve shear stress and cross web stress. Furthermore, these stresses, as well as machine direction stress, are often nonuniform. Some work has been done for particular cases using continuum mechanics software. But there are no two-dimensional models that capture the relevant physical principles in a way that can provide a general basis for calculation and insight. Some of the issues that might be addressed with such a model are:

- Localized loss of traction due to nonuniform stress
- The amount of spreading that can be supported on a concave or curved roller
- Strain transport into the next span
- Interaction of spans due to loss of traction on part of the roller

In this paper, the two-dimensional equations of equilibrium for a thin web on a roller are developed from first principles, taking into account cylindrical roller geometry and the effects of friction between the web and roller. The questions listed above are explored by experiment and FEA analysis. A method is developed for determining the conditions that must be met at the entry to a roller to insure that entry span stresses - machine direction, cross web or shear - do not cause slipping. Conditions for both nonuniform webs and nonuniform rollers are treated.

NOMENCLATURE

G	Shear modulus, Pa
h	Web thickness, m
L	Length of span, m
r	Roller Radius, m
u	Particle displacement in x direction, m

u_y Derivative of u with respect to y
v Particle displacement in y direction, m
v_x Derivative of v with respect to x
V_y Velocity in the y direction, m/s
V_s Velocity of web along axis of roller, m/s
V_u Surface velocity of upstream roller, m/s
V_d Surface velocity of downstream roller, m/s
γ_{xy} Elastic shear strain
ε Elastic strain
ε_o Longitudinal strain at entry of upstream roller
η Deformed y coordinate, m
φ_s Static coefficient of friction between web and roller
φ_d Dynamic coefficient of friction between web and roller
μ Poisson's ratio
ξ Deformed x coordinate, m
σ Stress, Pa
σ_r Stress normal to surface of web, Pa
τ_{xy} Shear stress in x,y plane, Pa
ψ Angle of tangent to particle trajectory of web (in relation to x-axis), radians

Subscripts

u Upstream
d Downstream
θ Polar angle in cylindrical coordinates
r Direction normal to roller surface
x Aligned with x-axis
y Aligned with y-axis
z Aligned with z-axis (normal to web plane)

INTRODUCTION

The importance of understanding roller traction has been appreciated from the earliest days of web processing and there is considerable literature on the subject. An excellent review can be found in a 2001 IWEB paper, "Traction in Web Handling: A Review" by Dilwyn Jones [1]. It is evident from the bibliography of this paper that little work has been done on the two-dimensional aspects of the problem. A notable exception is a 1995 paper by Zahlan and Jones, titled "Modeling Web Traction on Rollers" [2], in which they used continuum mechanics software (ABAQUS) to perform a parameter study. This enabled them to make some interesting qualitative observations about cross web effects.

An example of a problem that needs two-dimensional traction analysis is predicting when a web will slip on a spreader roller. For a problem like this, a method is needed that can evaluate traction incrementally across the web as it enters onto a roller.

Several of the experiments described in this paper were made at strains in excess of 10% on a latex web with a coefficient of friction close to 1.0. These are not presented as representative of actual processes. Their value is in making it possible to better visualize fundamental concepts.

TERMINOLOGY

The following terminology will be used in this paper

- Stick zone – An area on the roller where web and roller surface speeds are perfectly matched.
- Microslip zone – An area on the roller where motion of the web relative to the roller surface is caused entirely by variation in the web strain.

ASSUMPTIONS

1. The effect of air lubrication is accounted for in the coefficient of friction and some amount of Coulomb friction always exists between the web and roller.
2. The web is in a steady state of motion, that is, the paths of particles of the web (analogous to streamlines in fluid flow) are not changing shape.
3. In the stick zone, the static coefficient of friction is used. The dynamic value applies in the microslip zone.
4. Lateral web analysis generally requires the use of the nonlinear equations of elasticity. However, for the case of a web on a roller the linear equations of elasticity will usually be adequate because,
    a. For purposes of this analysis, the *x*-*y* coordinate system can be defined in relation to the roller rather than the process. In other words, the *y* coordinate can be assumed to be rotated so that it is parallel to the roller axis, even when the roller axis is not perpendicular to the process centerline.
    b. Item 3.a will put the roller coordinate system into alignment with the coordinate system used in nonlinear analysis to express the stresses and strains at the end of the span preceding the roller. So, it's a "natural" coordinate system for on-roller analysis.
    c. Changes in elastic rotations in the short segment of web on a roller will generally be insignificant.
    d. Even if a particular case requires a nonlinear formulation, it is an easy matter to substitute the expressions for nonlinear strain and stress into the linear formulation when they are needed.
5. The roller may have a nonuniform diameter. But, it has a straight axis.

PLANE STRESS DEFINITIONS

Displacements from the relaxed coordinates *x* and *y* are *u* and *v*, respectively. Strains are defined as follows.

Strain in the *x* direction $$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \{1\}$$

Strain in the *y* direction $$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \{2\}$$

Shear strain $$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \{3\}$$

$$u_y = \frac{\partial u}{\partial y} \qquad \{4\} \qquad\qquad v_x = \frac{\partial v}{\partial x} \qquad \{5\}$$

Strain in the *z* direction $$\varepsilon_z = \frac{-\mu}{1-\mu}(\varepsilon_x + \varepsilon_y) \qquad \{6\}$$

Deformed coordinates are

$$\xi = x + u \qquad \{7\} \qquad\qquad \eta = y + v \qquad \{8\}$$

Assuming Hook's Law, the stresses may be expressed in terms of strains, Poisson's ratio, μ, and modulus of elasticity, E, as follows.

The *x*-axis stress is: $$\sigma_x = \frac{E}{1-\mu^2}\left[\varepsilon_x + \mu\varepsilon_y\right] \qquad \{9\}$$

The *y*-axis stress is: $$\sigma_y = \frac{E}{1-\mu^2}\left[\varepsilon_y + \mu\varepsilon_x\right] \qquad \{10\}$$

The shear stress is: $$\tau_{xy} = \frac{E}{2(1+\mu)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \qquad \{11\}$$

The transformations from undeformed to deformed coordinates are

$$d\xi = \left(1+\varepsilon_x\right)dx + u_y dy \qquad\qquad d\eta = v_x dx + \left(1+\varepsilon_y\right)dy \qquad \{12\}$$

BOUNDARY CONDITIONS AT A DOWNSTREAM ROLLER WHEN A STICK ZONE EXISTS FROM EDGE TO EDGE OF THE WEB AT THE ROLLER ENTRY

Steady state boundary conditions at the entry to a roller are the normal entry and normal strain rules presented in the author's 2005 paper, "A New Method for Analyzing the Deformation and Lateral Translation of a Moving Web" [3].

Normal entry rule for a uniform web: $$\tan^{-1} v_x (1+\varepsilon_x)^{-1} \approx v_x = \theta_r \,. \qquad \{13\}$$

where θ_r is the angle of misalignment of the roller.

Normal strain rule: $$\frac{1+\varepsilon_x}{1+\varepsilon_o} = \frac{V_d}{V_u} \qquad \{14\}$$

where ε_x is the strain normal to the axis of the downstream roller, V_d and V_u are, respectively, the downstream and upstream roller surface velocities and it is understood that these may be a function of *y*. ε_o is the longitudinal strain at the entry to the upstream roller of the span and may also be a function of (*y* + *v*) if that roller is nonuniform.

GENERALIZED CONDITIONS FOR STEADY STATE FLOW OF AN ELASTIC SOLID

There are several fundamental relationships for steady state elastic flow that will be useful in analyzing web behavior on a roller. They are always true, regardless of the nature of the forces that cause web deformation.

The boundary conditions expressed in equations {13} and {14} are special cases of a general description of particle motion in an elastic solid. In the previously mentioned 2005 paper [3] the equation for the steady state trajectory of a particle in a deformed web was derived and it was shown that it makes an angle ψ with the MD direction defined by,

$$\psi = \tan^{-1}\frac{v_x}{1+\varepsilon_x} \qquad \{15\}$$

where ε_x and v_x are defined in equations {1} and {5} . Furthermore, the strain in the direction of the trajectory must satisfy the requirement for constant mass flow,

$$\frac{1+\varepsilon_\psi}{1+\varepsilon_o} = \frac{V_\psi}{V_o} \qquad \{16\}$$

where ε_ψ is the strain in the direction of the trajectory and V_ψ is the velocity along the trajectory. The strain ε_o and the velocity V_o are assumed to be known values at the entry to an upstream roller. Equations {15} and {16} must hold at every point in steady state motion of a web. The value of ε_ψ may be computed from values in the *x-y* coordinate system by the usual relationship,

$$\varepsilon_\psi = \varepsilon_x \cos^2(\psi) + \varepsilon_y \sin^2(\psi) + \gamma_{xy} \sin(\psi)\cos(\psi) \qquad \{17\}$$

EQUATIONS OF EQUILIBRIUM ON A ROLLER SURFACE:

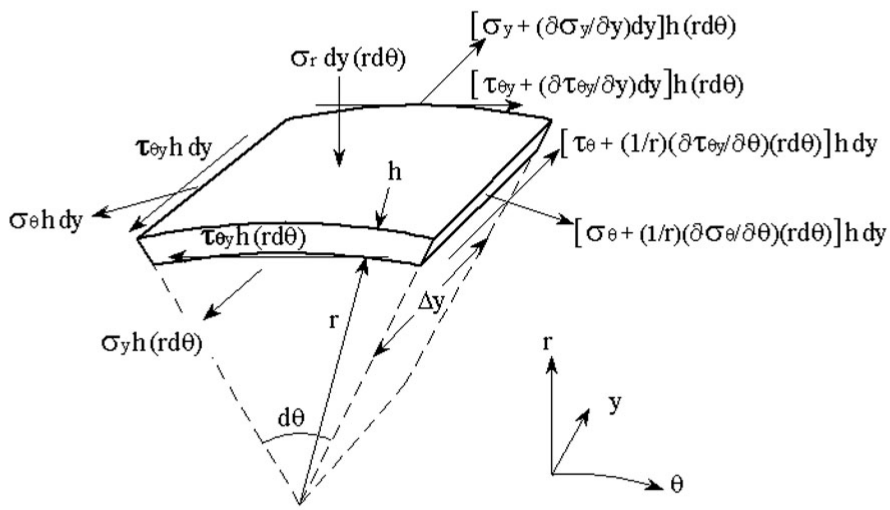

Figure 1
Forces on an infinitesimal element of web on a roller

Figure 1 illustrates the forces on an infinitesimal element of web on a roller in cylindrical coordinates. The web is assumed to be a thin membrane with no bending stiffness. Applying the usual method for developing the equations of equilibrium in two dimensions results in the following.

Equating forces in the θ direction,

$$\left[\left(\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} r\, d\theta + \sigma_\theta\right) h\, dy - \sigma_\theta\, h\, dy + \left(\frac{\partial \tau_{\theta y}}{\partial y} dy + \tau_{\theta y}\right) h\, r\, d\theta - \tau_{\theta y}\, h\, r\, d\theta\right] = F_\theta \qquad \{18\}$$

Equating forces in the *y* direction,

$$\left[\left(\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right) h\, r\, d\theta - \sigma_y\, h\, r\, d\theta) + \left(\tau_{\theta y} + \frac{1}{r}\frac{\partial \tau_{\theta y}}{\partial \theta} r\, d\theta\right) h\, dy - \tau_{\theta y} h\, dy\right] = F_y \qquad \{19\}$$

The terms on the right hand sides of these equations represent the forces of friction in the incremental area *rdθdy*. In a free span they will be zero. On the roller their values will depend on whether the web is slipping. Eventually, their direction and magnitude must be determined. But for now, let it suffice to say that frictional forces will exist on the roller.

Before proceeding further the normal stress, σ_r should be defined in terms of σ_x. Referring to Figure 1,

$$\sigma_r (r\,d\theta)dy = \sigma_x h dy 2\sin(d\theta/2) \qquad \{20\}$$

or

$$\sigma_r = \sigma_x \frac{h}{r} \qquad \{21\}$$

Simplification of coordinates

Cylindrical coordinates were used in the initial derivation of the equilibrium equations because they seemed appropriate for a roller. However, in the case of a thin membrane that remains in contact with a roller, it is possible to return to an *x-y* coordinate system. It should be noted that the ability to make this transformation suggests that when bending stiffness can be ignored, the cylindrical shape of the web on a roller imparts no special mechanical attributes to it.

Since r is constant, the product of r and θ is the simply the circumferential position. Thus, the cylinder can be treated as though it is unwrapped and $r\theta$ becomes a linear coordinate. This new coordinate will be called x and equations {18} and {19} become,

x-direction
$$\left[\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right] h = \frac{F_x}{dy\,dx} = S_x \qquad \{22\}$$

y-direction
$$\left[\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\right] h = \frac{F_y}{dy\,dx} = S_y \qquad \{23\}$$

The radial stress σ_r will, of course, still exist and the roller surface behind the web will be replaced by a fixed plane.

The quantities S_x and S_y may be thought of as frictional stresses. Now, the values of S_x and S_y will be addressed. There are two cases to consider, stick and microslip.

THE STICK ZONE

In a stick zone, the values on the left sides of equations {22} {23} are fixed at some value determined by prior conditions and the friction forces F_x and F_y adjust themselves so that the right sides exactly equal the values on the left, thus maintaining equilibrium. This condition is maintained so long as the vector sum of the forces opposing the friction is less than the maximum force of friction created by the normal stress, σ_r. Wherever this condition prevails, the web is effectively locked to the roller. So, using equation {21}, the condition for sticking is,

$$\sqrt{S_x^{\ 2} + S_y^{\ 2}} \le \frac{\varphi_s |\sigma_x|}{r} h \qquad \{24\}$$

where φ_s is the static coefficient of friction and F_x and F_y are defined by equations {22} and {23} (remember, the values on the left sides have become fixed).

$$S_x = h\left[\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right]_{fixed} \qquad \{25\}$$

$$S_y = h\left[\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\right]_{fixed} \qquad \{26\}$$

Regarding the vector sum in {24}: If one imagines an object, subject to two orthogonal forces and resting on a frictional surface, it is easy to see that the object begins to move when the vector sum of those forces exceeds the force of friction. Furthermore, the vector sum will always reach the point of slipping before either of the orthogonal components and, if the forces don't change, the motion will be in the direction of the vector sum.

<u>The conditions for sticking at the entry to a roller</u>

The conditions for a stick zone at the entry to a roller can now be defined. Before reaching the roller, the right sides of {22} and {23} are zero. So, equilibrium requires that the two derivatives on the left be equal and opposite. However, when the web arrives on the roller and is not slipping, there can be no variation of stress in the *x*-direction. As it enters onto the roller, the web can be imagined as a series of narrow strips parallel to the roller axis. If the web is in a steady state, each strip enters onto the roller in the same state as the one just before it. So, in the stick zone, the derivatives with respect to *x* disappear and are replaced by the force of friction. The equations of equilibrium then become,

$$\frac{\partial \tau_{xy}}{\partial y} = \left.\frac{\partial \tau_{xy}}{\partial y}\right|_{at\,entry} \qquad \{27\}$$

$$\frac{\partial \sigma_{y}}{\partial y} = \left.\frac{\partial \sigma_{y}}{\partial y}\right|_{at\,entry} \qquad \{28\}$$

Mathematically, the stick condition for each point across the web at the entry to a roller can now be stated as,

$$\sqrt{\left(\frac{\partial \tau_{xy}}{\partial y}\right)^2_{at\,entry} + \left(\frac{\partial \sigma_{y}}{\partial y}\right)^2_{at\,entry}} \le \varphi_s \frac{|\sigma_x|}{r} \qquad \{29\}$$

For convenience in discussion, the term on the left of {29} will be called the stress rate and the term on the right will be called the friction rate. The friction rate is always taken to be positive. The friction rate may be thought of as a measure of how much of the in-plane stress, σ_x, is converted to friction per unit of circumference).

It is tempting to think that the stick condition defined by {29} has something to do with the direction of motion when an increment of web starts slipping. For example, the magnitudes of the x and y stresses can be used to compute an angle that represents the direction of the force vector in the stick zone. But, this is only a <u>criterion</u> for <u>existence</u> of the stick zone. A completely different situation applies if microslip occurs at any location along the line of entry to a roller. Then, the stress field upstream of the roller will probably change and that in turn will alter the values in the stick criterion.

It is worthwhile to stop for a moment and reflect further on the change in the balance of forces that occur in a web as it enters onto a roller. Before the web reaches the roller, the right sides of {22} and {23} will be zero. This is because the two terms on the left occur entirely in the web and there are no other forces influencing equilibrium. In other words, when there is no friction, the spatial rate of change of σ_y in the *y*-direction must be equal and opposite to the spatial rate of change of τ_{xy} in the *x*-direction. When the web reaches the roller, the normal entry and normal boundary conditions operate by imposing constraints on the deformation. If there is to be no slipping (a stick zone) then the *x*-derivatives must be zero and any *y*-derivative terms must be balanced entirely by traction.

An example of the use of this analysis is based on test data taken from a 1999 IWEB paper by Good and Straughan [4]. The subject of the paper was wrinkling caused by web twist. If the tension was low, it sometimes took much more twist (as much as two times more) to create a wrinkle than at higher tensions. The authors suggested that low tension led to low traction and that this allowed troughs to flatten on the roller surface rather than develop into wrinkles. This conclusion is supported by Figure 2, which shows a plot of the stress rate and friction rate for a particular case. The calculations for this case and for those illustrated in Figures 3 through 7 were made with an FEA model described in a paper [5] presented by the author at the 2007 IWEB conference.

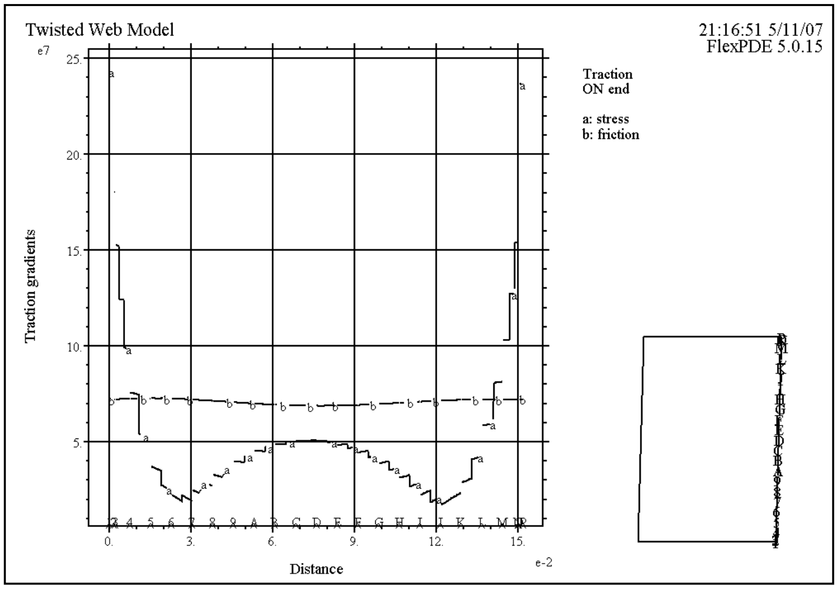

Figure 2

Stress rate (a) and friction rate (b) for a twisted web

Twist = 5 degrees. length = 0.108 m, width = 0.152 m, thickness = 23.4e-6 m, modulus = 4.13e9 Pa, tension = 26.7 N, μ = 0.3, roller diameter = 0.0736 m

At any location where the stress rate is larger than the friction rate, local slipping may occur. The friction rate in Figure 2 is based on a friction coefficient of 0.3. It is evident that the edges can slip and that it would take only a slight change in either the friction coefficient or the stress rate for any of the other points to slip. In all of the other cases where this behavior was observed, the FEA analysis produced data like Figure 2 or worse (with the friction curve below the stress curve). In the cases where “normal” behavior was observed, there was ample separation between the stress and friction rate curves.

<u>Application of the stick criterion to a concave spreader</u>

The stick criterion can be used to decide whether a concave roller can spread a web without slipping. The following example illustrates a typical result. The material is polyethylene with a modulus of 50,000 psi, Poisson ratio of 0.35 and 0.001 inch thickness. The span is 20 inches long and 60 inches wide. The roller is 72 inches long. The diameter is 6 inches and the profile is circular with a depth at the center of 0.05 inch. Results at three tensions are illustrated, 0.5, 1 and 2 pli. Note: the vertical scales of the three graphs are different.

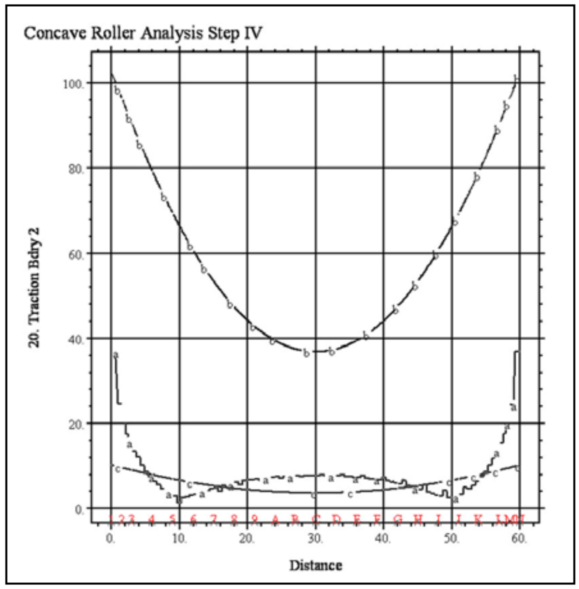

Figure 3
Stick criterion at 0.5 pli
a) stress rate b) friction rate c) friction rate, φs = 0.035

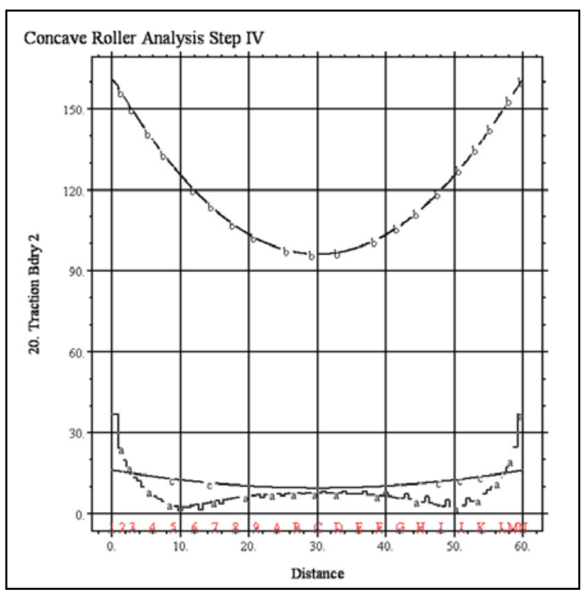

Figure 4
Stick criterion at 1.0 pli
a) stress rate b) friction rate c) friction rate, $\varphi_s = 0.035$

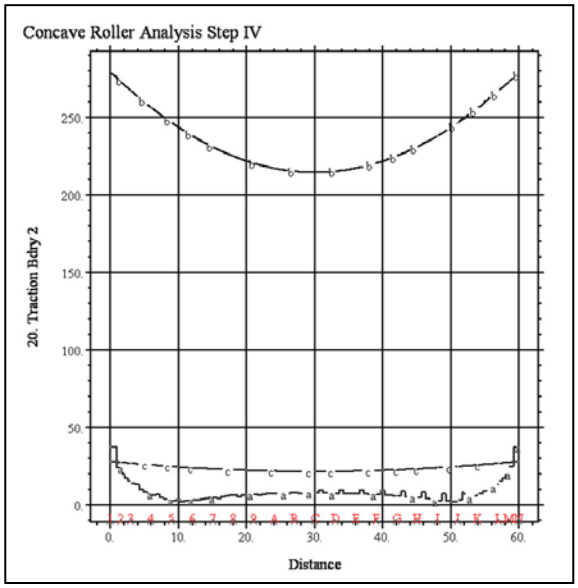

Figure 5
Stick criterion at 2.0 pli
a) stress rate b) friction rate c) friction rate, $\varphi_s = 0.035$

Two friction rates are shown in the graphs. One is for a coefficient of friction equal to the assumed static value of 0.35 and the other is for 10% of that value. The web won't slip laterally at any point where the friction rate exceeds the stress rate. So, at speeds where air lubrication is not a factor, the traction is adequate in all three cases. But, at line speeds high enough to affect friction, slipping can occur. In Figure 3, where the tension is lowest, a coefficient equal to one tenth of the static value will almost disable the spreading. Increasing the friction rate by increasing MD tension restores it. In Figure 5, only narrow zones at the edges still have a problem.

Friction rate is directly proportional to the coefficient of friction and MD tension (at a given cross web location). So, for a given roller, worst case is low tension and high line speed.

Application of stick criterion to a cambered web

An example of the stick criterion applied to a cambered web is shown in **Figure 6**. This example was taken from a series of experiments reported by Swanson in 1999 [6]. The material is PET with a modulus of 3.45 GPa, Poisson ratio of 0.35 (assumed), 23.4 micron thickness and 139 m radius of curvature. The span is 2 m long and 0.305 m wide. Average tension is 66 N. Roller diameter is 7.6 cm. Coefficient of friction is 0.20. Assuming the roller was free turning with ample wrap, there was little chance of the web slipping. The average tension in this test was well above the critical tension necessary to avoid a slack edge (22 N).

Another test in the Swanson series is shown in **Figure 7**. In this case the average tension was at the critical value of 22 N. [There was probably no negative tension in the experiment. The experimenter would have noticed it. Futhermore, the calculated average tension in the FEA model was 21.76 N.] All of the other parameters were the same except for span length of 0.67 m and the radius of curvature of 185 m. As would be expected, the friction rate drops to zero at the edge where the MD tension is zero. This would obviously not be a good production situation where tension and friction could vary.

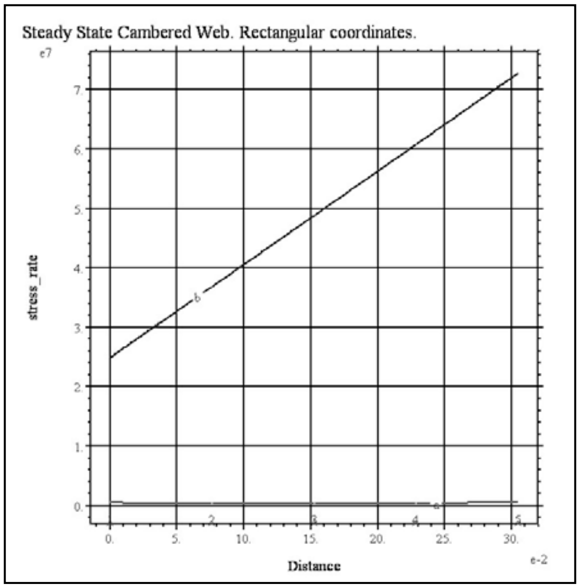

Figure 6
Stick criterion for a cambered web, high tension

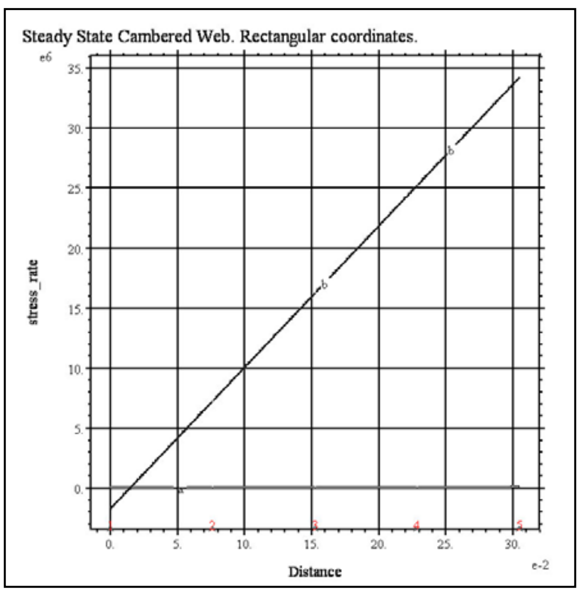

Figure 7
Stick criterion for a cambered web, critical tension

<u>What happens if a web doesn't satisfy the stick criterion?</u>

If equation {29} isn't satisfied on some parts of the contact line at the roller entry, then, at those places, the boundary conditions {13} and {14} can't be met and the web must slip there. It is natural to think that maybe the web scuffs around a bit in a narrow zone at the entry and that farther downstream finds a new stress state that allows it to satisfy the boundary conditions at all points without slipping. That may be true. Slipping at the entry may alter the stress local stress patterns sufficiently to so that the normal entry and normal strain rules can operate farther in. However, it isn't clear that this will happen. For example, consider the case of a concave roller. The reason the web spreads on such a roller is that this is the only way it can deform so that it simultaneously satisfies the normal entry and normal strain conditions. The normal entry rule ensures that in the steady state there will be no further lateral motion without slipping and the normal strain rule ensures that the mass flow rate at every point across the span is constant. If these boundary conditions can't be satisfied over some portion of the line of entry (for example at the edges), how can they be satisfied farther onto the roller? The law of conservation of mass must prevail and if the web can't move laterally to satisfy it, then it seems likely that there will be a local change in MD velocity of the web relative to the roller surface all the way around the roller. In other words, it will slip to satisfy conservation of mass.

Circumferential slipping may be occurring in Figure 8 below, which shows a 26-mil latex web being spread on a concave roller. This is a test in which everything has been pushed to extremes so that strains become visible to the unaided eye. The horizontal grid lines, which were straight in the relaxed web, curve upward with increasing distance from the web centerline. That is consistent with the MD stress profile to be expected of a concave roller. However, notice that at the very edges, in the circled areas, the horizontal grid lines flatten out. This is a possible indication of slipping in a situation, similar to the plot in Figure 2, where the stress rate has exceeded the friction rate at the edges. That is where slipping could be expected to start. This was a 180-degree wrap and the flattening of the curves was still observable as the web exited the roller.

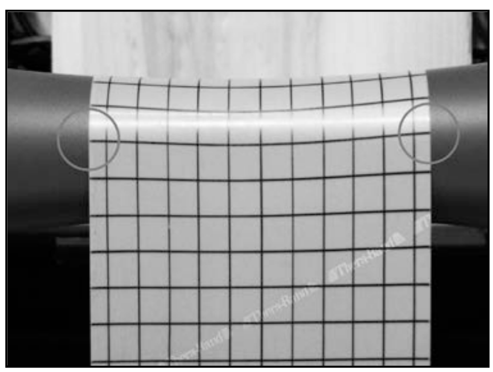

Figure 8
Slipping on a concave roller

THE MICROSLIP ZONE

For a web with a stick zone, a microslip zone <u>must</u> develop if there is a tension difference across the roller. It is the drag of the microslip that establishes equilibrium between the web forces and the roller torque.

Since the web will be moving relative to the roller in the microslip zone, it is clear that the force of friction will be oriented so as to oppose the local web motion. The equations of equilibrium should look like {22} and {23}. But the friction in the direction of motion will always be equal to the maximum value. So, equation {24} will now be,

$$\sqrt{S_x^{\,2} + S_y^{\,2}} = \frac{\varphi_d\ \sigma_x}{r} h \qquad \{30\}$$

where φ_d is the dynamic coefficient of friction. There is now the question of direction. That is answered by equation {15}, which provides the angle of direction of particle motion relative to the *x*-axis. Using this information, the equations of equilibrium for the microslip zone become,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \frac{\varphi_d\ \sigma_x}{r}\cos(\psi) \qquad \{31\}$$

and

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \frac{\varphi_d\ \sigma_x}{r}\sin(\psi) \qquad \{32\}$$

Since MD stress is always much larger than lateral stresses, it seems reasonable to assume that the signs of the right hand friction terms will depend only on whether the roller is driving or braking.

This looks like a wonderful result. Equations of equilibrium have now been rigorously developed for both the stick and microslip zones. Unfortunately, they can't simply be plugged into a pseudo-static FEA model. The reason is illustrated by what happens when a length of thin film is draped over a roller with weights on each end to produce tension. A microslip zone develops on both sides of the roller. As the roller is turned in one direction or the other, the upstream microslip zone is consumed until only a single, longer one exists at the downstream end. Furthermore, as the ABAQUS model in the Zahlan and Jones paper [2] showed, the transition line between the stick and slip zones is not straight. This is obvious to anyone who has watched a web guide change the tension distribution at the exit of an upstream roller.

Figure 9
A static friction test

So, something more is needed for a pseudo-static model. That will have to wait for a future paper.

<u>Importance of the microslip zone for nonuniform webs and rollers</u>

Understanding the microslip zone is the key to a number of issues.

1. It is the zone in which torque is transferred between the web and roller.
2. It is the zone where stresses transferred from the previous span have their first effect. This may be especially important for nonuniform webs.
3. Nonuniform stress downstream of a roller can cause part of a microslip zone to extend all the way to the line of entry, thus invalidating assumptions that permit pseudo static analysis of single spans. This can probably happen before a web edge goes slack.

SOME OBSERVATIONS OF WEB BEHAVIOR ON ROLLERS

Troughs at the exit of a driven roller

It is common knowledge that troughs can form at the exit of driven rollers. The web must expand laterally (reduction of Poisson contraction) when moving from an area of higher to lower tension. Friction between the web and roller constrains the web laterally, producing compressive stress that buckles the web immediately following its exit from the roller surface. Figure 10 shows a nice example of this. The web is 26 mil latex and the angle of wrap is 155 degrees. The coefficient of friction for this combination of materials ranges from 0.75 to 1.0 depending on cleanliness (wiping the web and rollers with isopropyl alcohol will produce the higher value for several runs of the 25 foot web). In this instance, it was probably close to the 0.75 value.

In this test, the web was being pulled at constant speed and the input tension was held constant as the roller drive torque was varied. At low values of torque (entry and exit tensions close to the same value) the troughs didn't form. As the drive torque was increased, there would be a value where the troughs were deepest. Then, as the torque was increased further, the exit tension (and the normal force that creates friction) would become low enough that the compressive stress could dissipate while the web was still on the roller surface.

It seems unlikely that troughs like this could turn into fully formed wrinkles, because the material where they could develop is always being moved off the roller. However, wrinkles could develop on a roller immediately downstream.

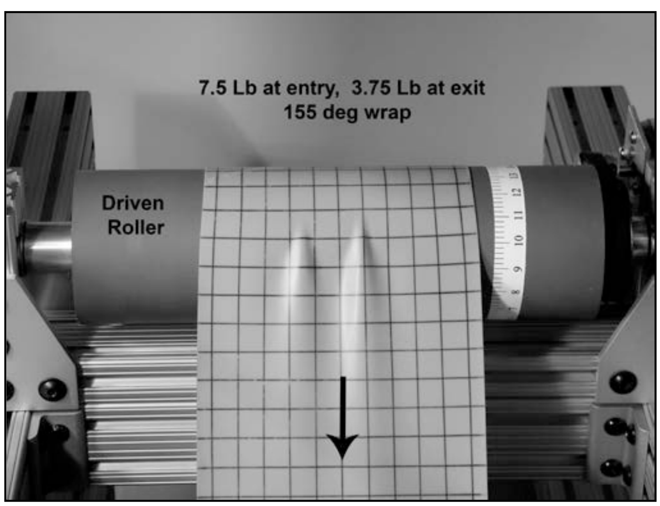

Figure 10
Troughs at exit of a driven roller

Wrinkles originating at the entry of a roller

A wrinkle study [7] was inspired by an excellent experiment described in the 1999 IWEB paper referenced earlier [8]. Twist provides a good way to study wrinkle formation because it produces only one or two MD wrinkles that maintain their lateral position. Furthermore, formation is easily controlled by adjusting the twist angle. The test setup is shown in Figure 11.

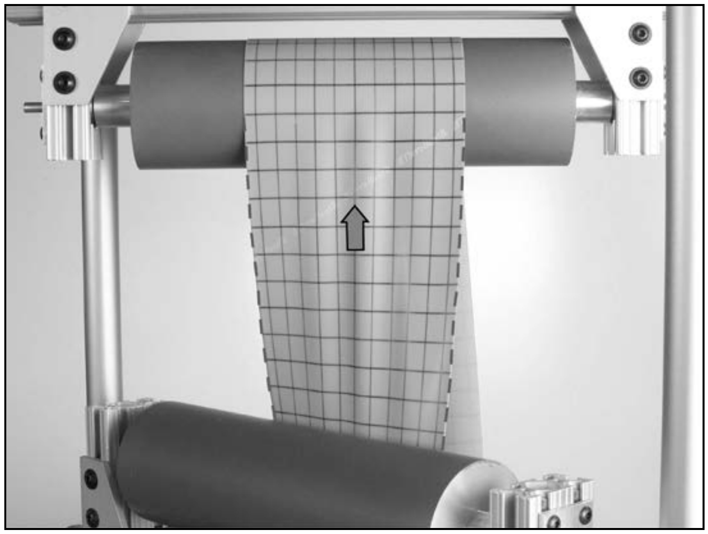

Figure 11

Twisted web at beginning of test

Operating tension is 6 lbf. The web is 26 mil latex with a modulus of 240 psi. The twisted span is 8.5 inches long and 5.5 inches wide. Wrinkles form at a twist angle of 45 degrees. Figure 11 shows the web in its twisted position before it begins moving in the machine direction.

The view in Figure 11 is perpendicular to the axis of the twisted roller at the top. The dashed red lines trace out the web edges. They appear to taper from full width at the top to a narrower value at the bottom. At the very bottom and top of the span, the web width hasn't changed. So, most of the taper is an illusion due to the angle of view. However, the taper near the top, where the web is viewed head-on, is real. This is because the edges have come closer together midway down the span. That is why the web has buckled. This is easy to understand if you imagine the web is gone and the dashed red lines are strings. Then, if the roller kept twisting, the strings would keep getting closer until they would overlap and make contact at 180 degrees of twist. Thus, the geometry of the twist creates compressive CD stress. The MD lines that were perpendicular to the roller axis before twisting have become slightly inclined toward the edges. This will bring the normal entry rule into play.

In Figure 12 the web has been allowed to advance in the machine direction. The troughs now begin to concentrate near the midline of the web. As a reference, a horizontal line has been superimposed on the photo just beyond the point of roller contact. Note that the horizontal black grid line is slightly curved in a "frown". This indicates that the tapered web geometry has begun to advance onto the roller. This brings the normal entry rule into play and, the web on both sides of the midline will track toward the center bringing excess material into the center of the web, increasing the CD compression. This behavior was postulated by Good, Kedl and Shelton in their paper, "Shear Wrinkling in Isolated Spans" [4]

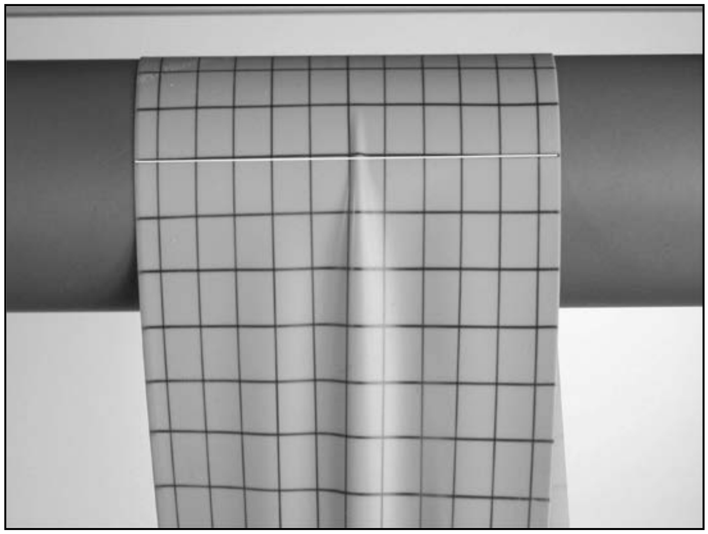

Figure 12
Beginning of wrinkle formation

If the angle of twist is adjusted just below the threshold of wrinkle formation, the condition in Figure 12 comes and goes. It appears that forces are competing to lift and flatten the web. The trough provides an initial lift at the line of entry. Then, if the combination of radial pressure and bending stiffness can't keep it flat, the lifted zone propagates around the roller.

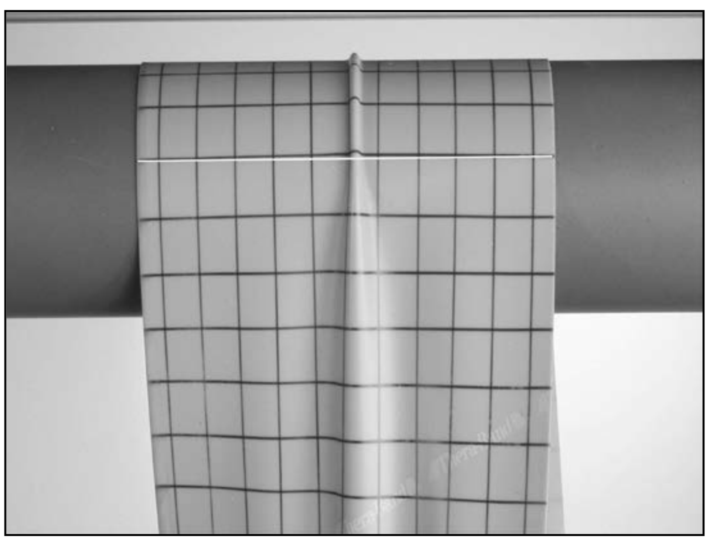

Figure 13
Fully formed wrinkle

The wrinkle in Figure 13 has now advanced completely through the wrap angle. It will continue to grow in height until it folds over. In this particular case, because of the extreme geometry, even the fold will continue to grow.

CONCLUSIONS

A general model for steady state flow of an elastic web has been described.

Equations of equilibrium, including friction, for a web on a roller (straight but not necessarily uniform) have been developed from first principles.

It has been mathematically demonstrated that if a web on a roller is flexible enough to be treated as a membrane and remains in contact with a roller, it may be treated as though it is flat. It should be noted that the ability to make this transformation suggests that when bending stiffness can be ignored, the cylindrical shape of the web on a roller imparts no special mechanical attributes to it.

A two-dimensional criterion has been established for the existence of a stick zone at the entry to a roller. This criterion is, in effect, a mathematical definition of the stick zone in terms of elasticity theory.

Applications of the stick criterion to concave rollers and a cambered web have been illustrated.

Equations of equilibrium for the microslip zone have been developed. But, it is not yet clear how to incorporate them into a comprehensive model.

Photographs illustrating wrinkling at both the entry and exit of a roller have been presented.

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