

The Effect of Mass Transfer on the Lateral Dynamics of a Uniform Web

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IWEB 2017

Introduction

- This paper starts where I left off with a paper delivered at the last IWEB Conference.
- In it, I attempted to develop a dynamic multi-span model that incorporated shear.
- At the end, I said,
- “A YSK-type Timoshenko model has been developed, and it looks quite plausible. However, it produces a value for the curvature factor that doesn’t make sense. After exhaustive troubleshooting, I’ve concluded that the problem is most likely something of a conceptual nature.”

Introduction

- I was right about that.
- And last summer, it finally dawned on me what it was.
- The effect of longitudinal strain on the rate of mass flow from one span to the next has been overlooked.
- This phenomenon is fundamental to the analysis of tension, but we have been missing its role in lateral dynamics.

Introduction (Cont.)

- We all know about the transport of strain equation,

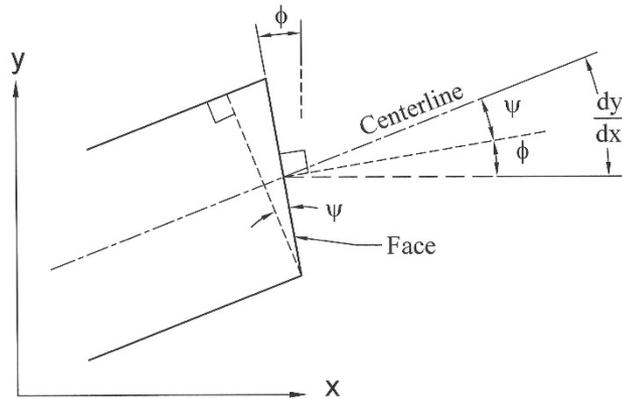
$$\frac{V_2}{V_1} = \frac{1 + \varepsilon_2}{1 + \varepsilon_1}$$

- The subscripts 1 and 2 refer to a span and its downstream companion. V and ε refer to the average velocities and strains in the spans.

Introduction (Cont.)

- It turns out that something similar is behind the acceleration equation - one of the two equations used in lateral dynamics to convert web shape to lateral motion (the other is the normal entry equation).
- I'm going to show you how it can be derived entirely from the same considerations of mass flow used in tension analysis.

First, some terminology



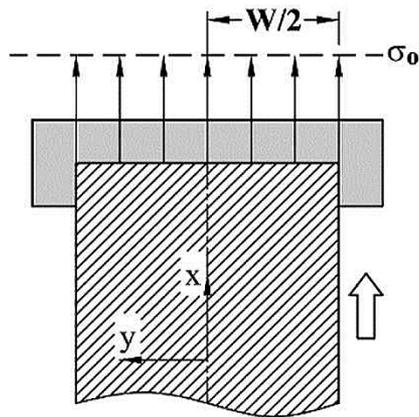
$$\frac{dy}{dx} = \phi + \psi$$

ϕ has two names – face angle or bending angle, depending on whether we're talking about the downstream face or an interior cross-section. In both cases, it refers to the change in angle (relative to the y -axis) of a plane that was perpendicular to the beam centerline in its relaxed state.

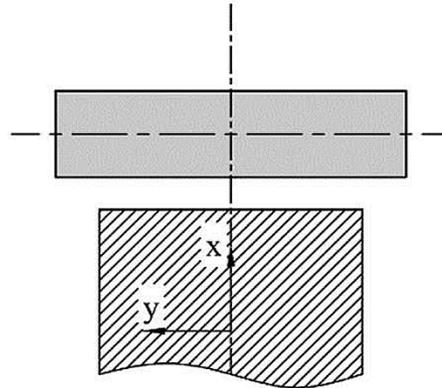
ψ is the shear angle.

The mass transfer idea

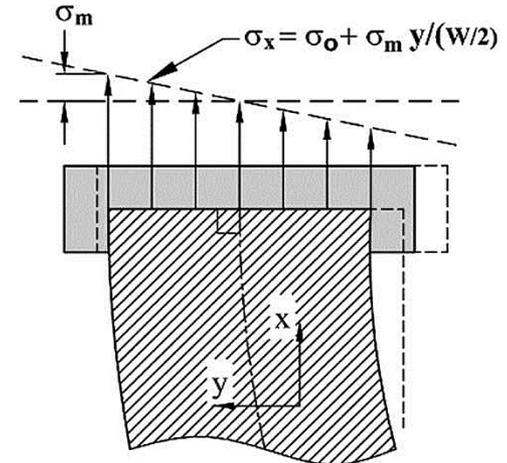
- We'll start with the simplest beam model, one without shear deformation – the tensioned Euler-Bernoulli (E-B) model.
- The next slide shows what happens when a moment exists at a downstream roller.
- The sequence starts with a web that is running in a steady state between parallel rollers.



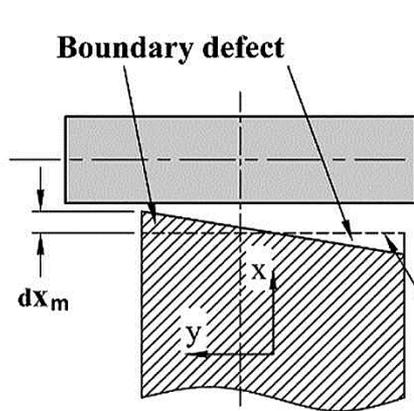
(a) Running web



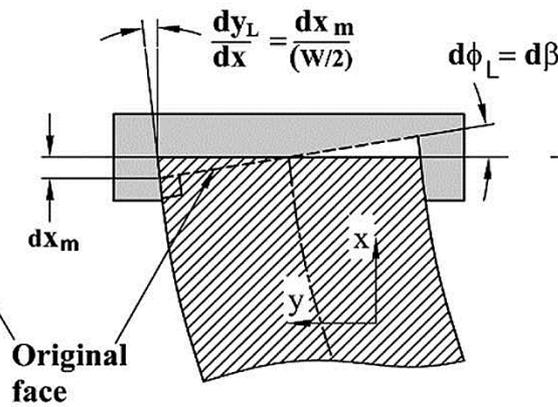
(b) Relaxed web



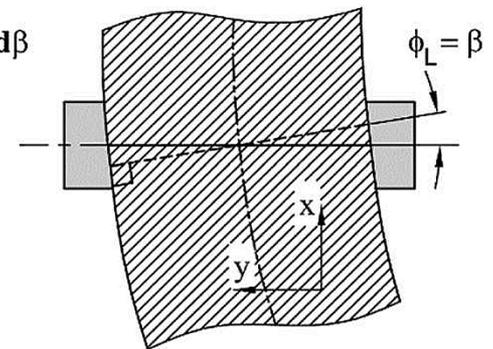
(c) Roller shifted to produce moment



(d) Effect of mass transfer on relaxed shape after time dt



(e) Altered web returned to the line of contact.



(f) Complementary effect on upstream end of next span

Incremental analysis of mass flow

- It's easy to show how the angle β (boundary defect) depends on strain at the web edge.

$$\frac{d\beta}{dt} = \frac{dx_m}{dt} \frac{1}{W/2} = \frac{V_o \varepsilon_m}{W/2}$$

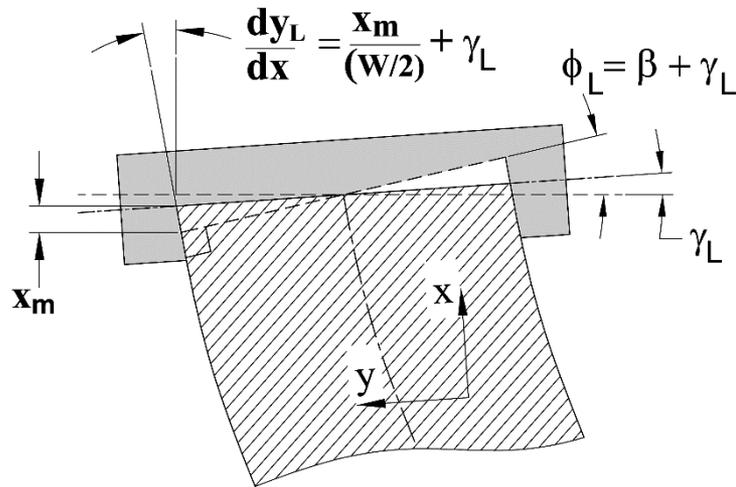
- Furthermore, the strain at the edge depends on curvature.

$$-\frac{\varepsilon_m}{W/2} = \frac{d^2 y_L}{dx^2}$$

- So,
$$\frac{d\beta}{dt} = -V_o \frac{d^2 y_L}{dx^2} = -V_o \frac{M}{EI}$$

Incremental analysis of mass flow

(Cont.)



So, now we know,

$$(1) \quad \frac{d\beta}{dt} = -V_o \frac{d^2 y_L}{dx^2}$$

$$(2) \quad \phi_L = \frac{dy_L}{dx} = \beta + \gamma_L$$

Taking the time derivative of (2) and substituting (1).

$$(3) \quad \frac{d}{dt} \phi_L = -V_o \frac{d^2 y_L}{dx^2} + \frac{d\gamma_L}{dt}$$

This is the mass transfer equation for webs with negligible shear deformation. It leads to the acceleration equation.

Incremental analysis of mass flow

(Cont.)

- Here is how it happens.
- Since $\phi_L = dy_L/dx$ for an E-B beam.

$$\frac{d}{dt} \phi_L = \frac{d}{dt} \frac{dy_L}{dx} = -V_o \frac{d^2 y_L}{dx^2} + \frac{d\gamma_L}{dt}$$

- The cross derivative is eliminated by taking the time derivative of the normal entry equation. The normal entry equation is,

$$\frac{dy_L}{dt} = V_o \left(\gamma_L - \frac{dy_L}{dx} \right) + \frac{dz_L}{dt}$$

- It defines the lateral velocity of the web on a roller.
- The term in parenthesis is called the entry angle. The last term is the lateral velocity of the roller itself.

Incremental analysis of mass flow

(Cont.)

- The result is the acceleration equation that Shelton used in his E-B dynamic model.

$$\frac{d^2 y_L}{dt^2} = V_o^2 \frac{d^2 y_L}{dx^2} + \frac{d^2 z_L}{dt^2}$$

- Also,

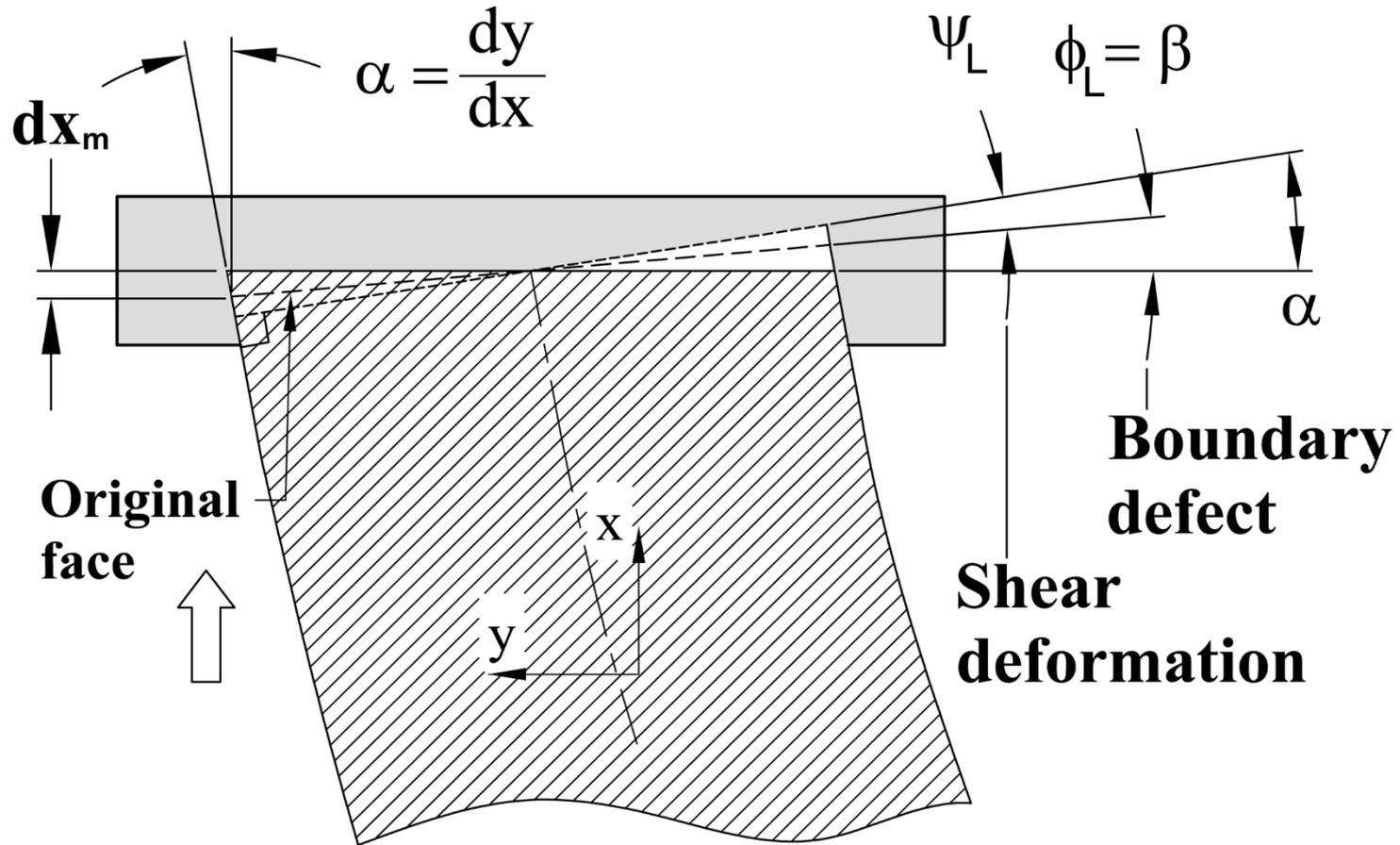
$$\frac{dy_L}{dx} - \gamma_L = \beta$$

- So, the normal entry equation is,

$$\frac{dy_L}{dt} = V_o (-\beta) + \frac{dz_L}{dt}$$

- Thus, if there is no boundary defect, the web doesn't move.

Mass flow with shear



Mass flow with shear (Cont.)

- From the Timoshenko beam equations,

$$M = aEI \frac{d^2 y_L}{dx^2} \quad \text{where} \quad a = 1 + \frac{nT}{AG}$$

- T = tension, A = cross-sectional area, G = shear modulus and n = shear coefficient.
- So, the time derivative of β is,

$$\frac{d\beta}{dt} = -aV_o \frac{d^2 y_L}{dx^2}$$

Mass flow with shear (Cont.)

- Since,

$$\phi_L = \beta + \gamma_L$$

- the mass transfer equation for the Timoshenko model is,

$$\frac{d\phi_L}{dt} = -aV_o \frac{d^2 y_L}{dx^2} + \frac{d\gamma_L}{dt}$$

- In a Timoshenko model, slope isn't equal to face angle. So, to get to the acceleration equation, an expression relating them is needed.

Acceleration equation (Cont.)

- The necessary equation comes from an analysis of the effect of geometric boundary conditions on beam shape. Details are in the paper. Here is the equation.

$$\frac{dy_L}{dx} = (y_0 - y_L) \frac{h_1}{L} + \phi_L h_2 + \phi_0 h_3$$

- h_1 , h_2 and h_3 are constants.
- Taking the time derivative and again using the normal entry equation to eliminate the cross derivative yields,

Acceleration equation (Cont.)

- The acceleration equation for models with shear – short, wide webs.

$$\frac{d^2 y_L}{dt^2} = aV_o^2 \frac{d^2 y_L}{dx^2} h_2 + V_o \left[\frac{dy_L}{dt} \frac{h_1}{L} - \frac{d\phi_o}{dt} h_3 + \frac{d\gamma_L}{dt} (1-h_2) - \frac{dy_o}{dt} \frac{h_1}{L} \right] + \frac{d^2 z_L}{dt^2}$$

- When $a = 1$, then $h_2 = 1$, $h_1 = h_3 = 0$, it defaults to the E-B acceleration equation, as it should.
- It passes all three of the validity tests described in the paper.
- When all of the time related terms are zero, it becomes the 4th boundary condition for the Timoshenko steady state model – zero curvature at $x = L$.

The meaning of β

- Since,

$$\phi_L = \frac{dy_L}{dx} - \psi_L \quad \text{and} \quad \phi_L = \beta + \gamma_L$$

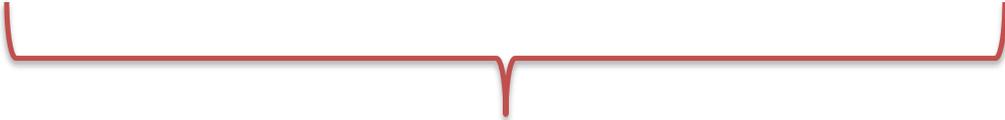
- the normal entry equation is,

$$\frac{dy_L}{dt} = V_o \left(-\beta - \psi_L \right) + \frac{dz_L}{dt}$$

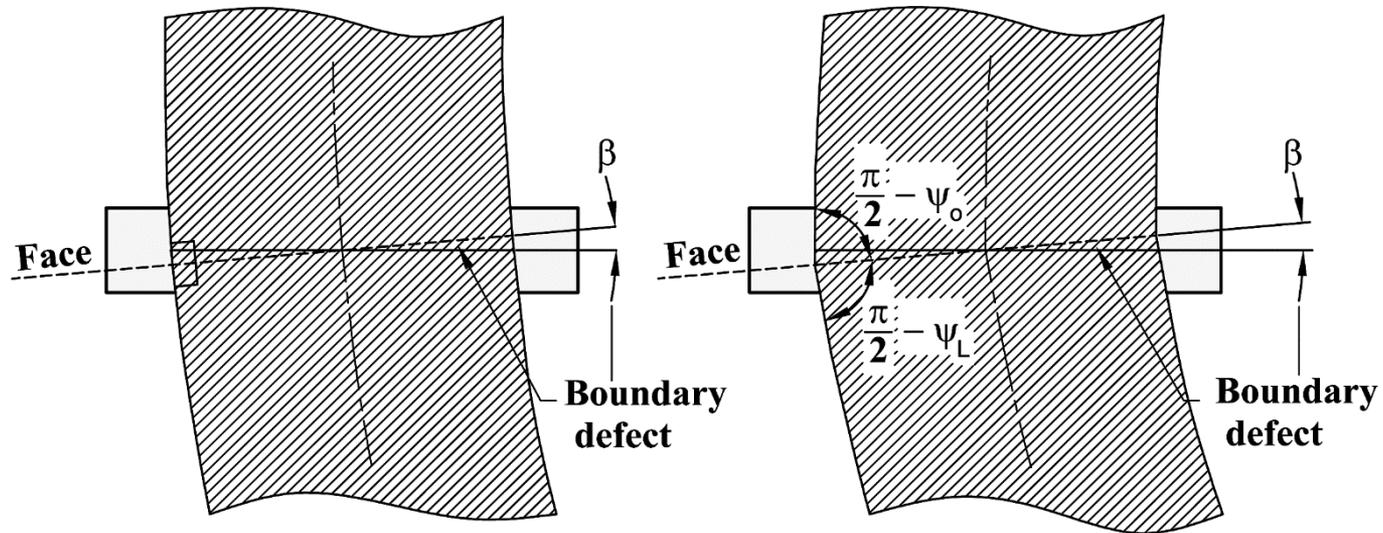
- Taking the time derivative of this will produce the acceleration equation of the previous slide.

The meaning of β (Cont.)

- In other words, the Timoshenko acceleration equation breaks down like this,

$$\frac{d^2 y_L}{dt^2} = aV_o^2 \frac{d^2 y_L}{dx^2} h_2 + V_o \left[\frac{dy_L}{dt} \frac{h_1}{L} - \frac{d\phi_o}{dt} h_3 + \frac{d\gamma_L}{dt} (1-h_2) - \frac{dy_o}{dt} \frac{h_1}{L} \right] + \frac{d^2 z_L}{dt^2}$$

$$= V_o \left(-\frac{d\beta}{dt} - \frac{d\psi_L}{dt} \right)$$

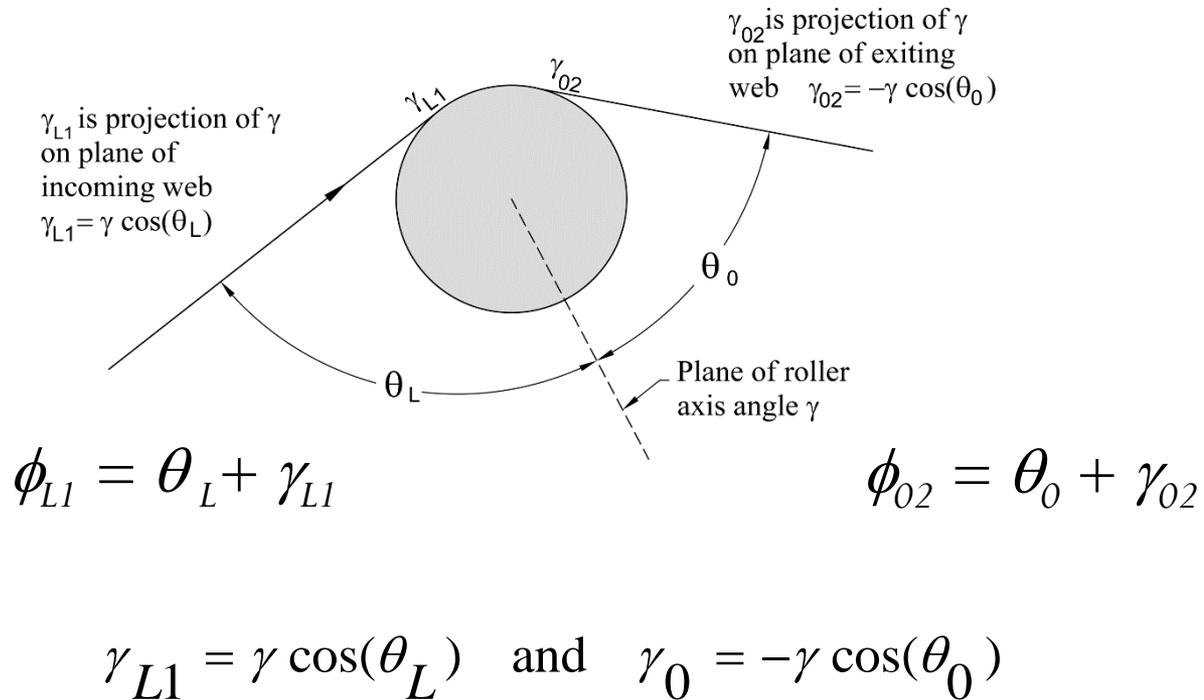
Complementary effect on next span



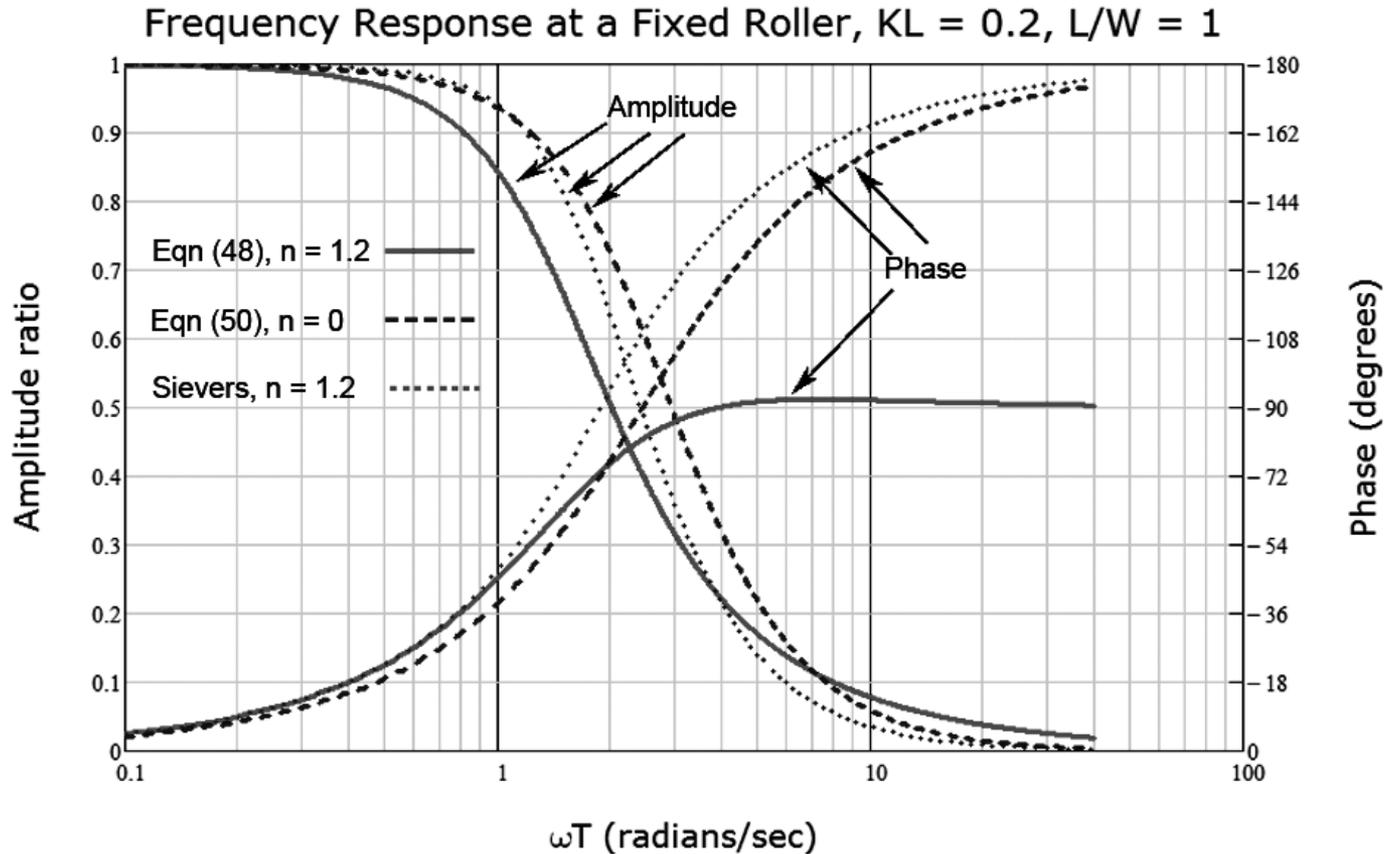
(a) Net effect of boundary defect on both spans, E-B model

(b) Net effect of boundary defect on both spans, Timoshenko model

Effect of roller axis angle on face angle

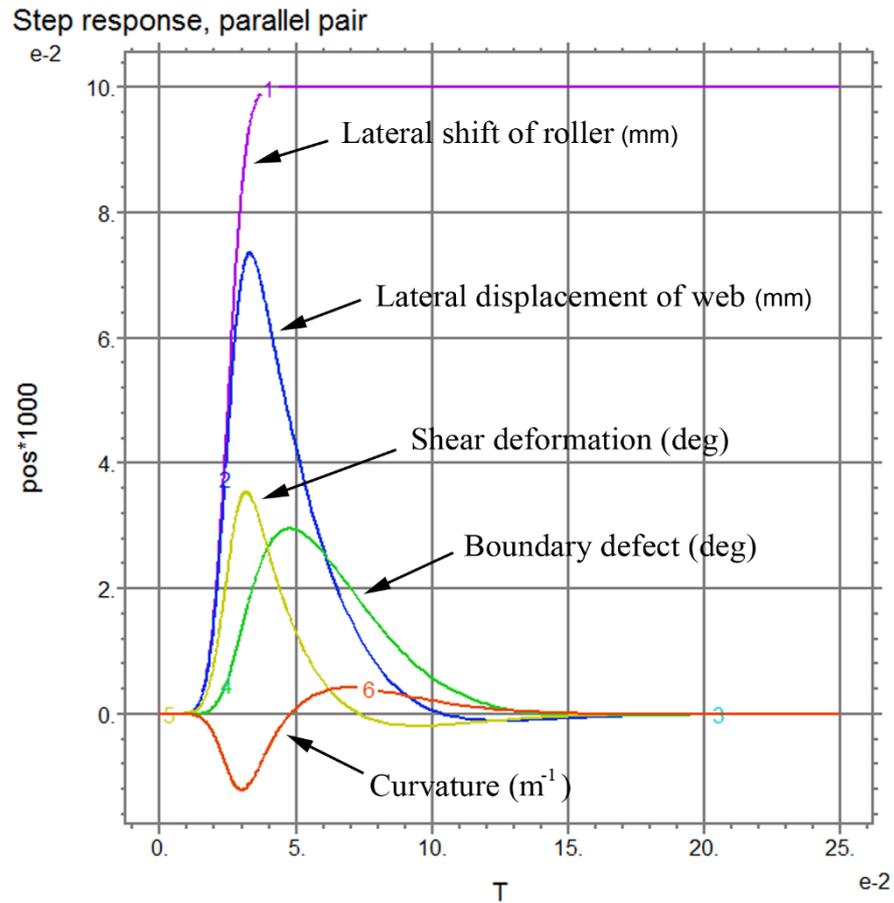


Results for $L/W = 1$

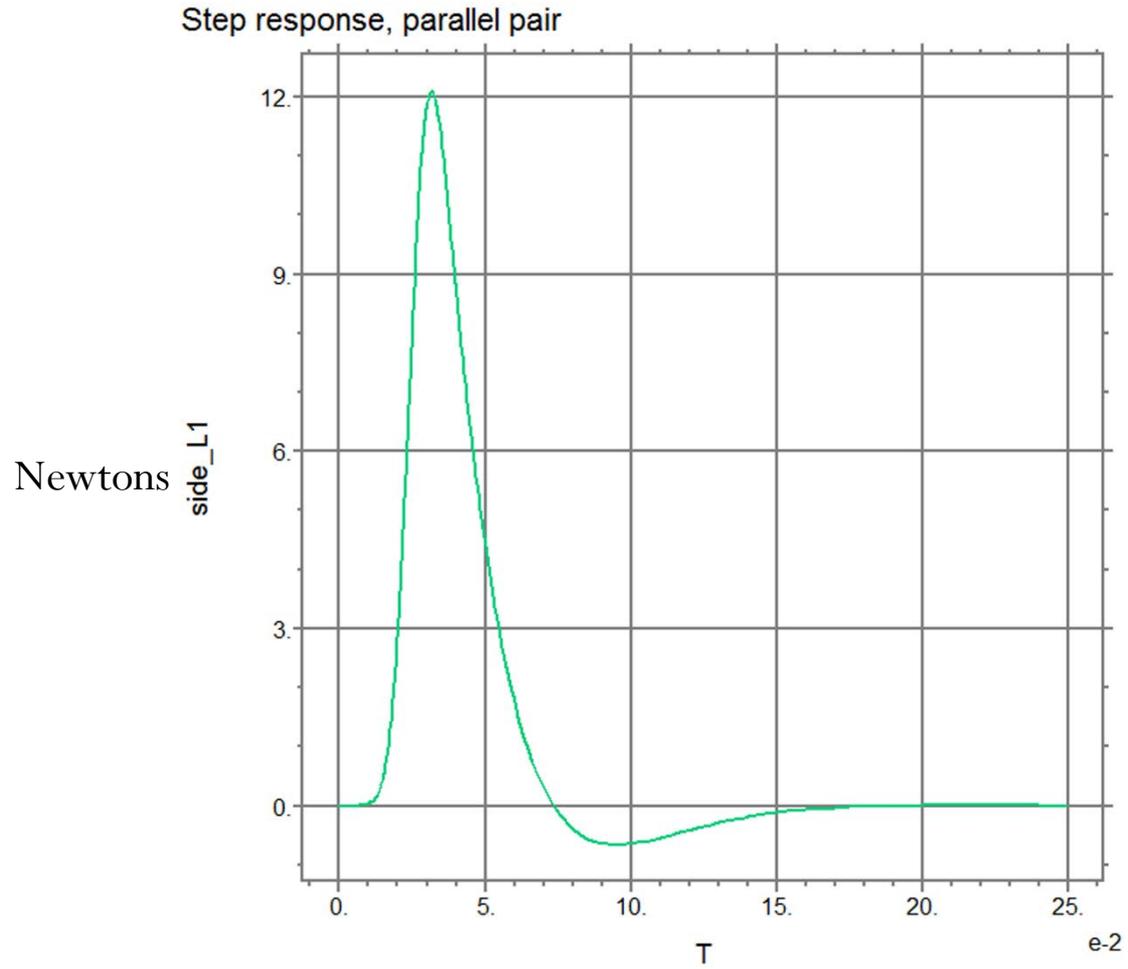


$KL = 0.2$, $L = 3$ inches, $T = 46$ *Lbf*, $W = 3$ inches, $h = 0.009$ inch,
 $E = 510,000$ psi.

Step response to roller shift at downstream end of parallel pair



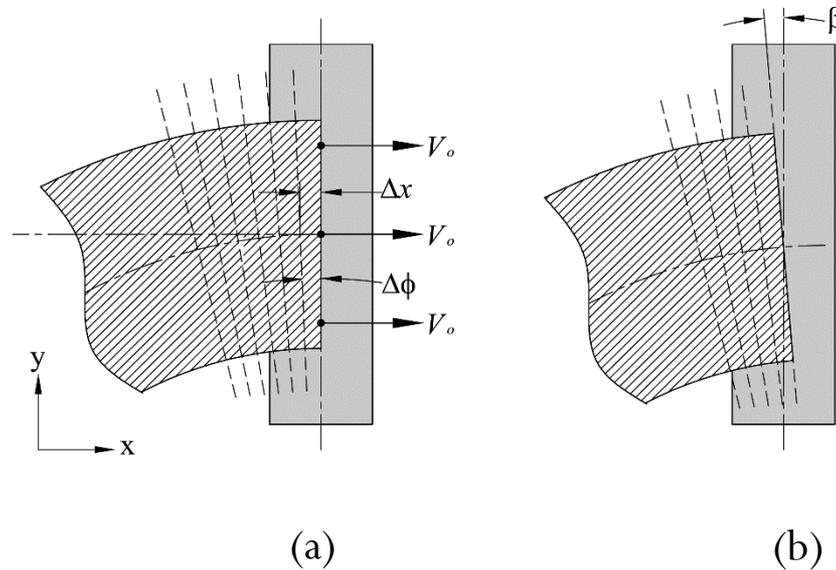
Side force



Benson's velocity matching criterion

Richard Benson suggested that the velocity equation, “is the result of imposing a velocity match between the downstream roller and the centerline of the web.” He then suggested that, “It is further expected that the web will stick to the roller for all points of first contact. – not just at the web's centerline. To achieve that, we must also match the rotational velocities of the roller [axis] and web 'face.'”

Benson's velocity matching Criterion (Cont.)



The diagram (a) shows a web immediately after it is bent by a lateral shift. The dashed lines are cross-sectional planes that define the bending angle f . Diagram (b) shows the web after a short time interval, dt .

Acknowledgements

- Thanks to Sinan Muftu who, after hearing my presentations at the last IWEB conference, drew my attention to Richard Benson's work.
- Also, thanks to Dilwyn Jones who was kind enough to review and critique this paper as it was being written. He made many helpful suggestions and is the one who discovered that Benson's acceleration equation is equivalent to the acceleration equation of this paper (even though it looks completely different).