### **Seeing the Invisible:**

### The Deformations and Stresses That Move Webs and the Two Rules That Govern Them

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### The invisibility problem

- Stress and strain are not directly visible.
- Deformation theoretically visible. But, usually too small to see.
- So, web process troubleshooting is done like internal medicine.
  - Observe the consequences and then use science to sort out the causes.
- The purpose of this paper is to:
  - Improve understanding of the science of lateral web behavior by making the deformations visible.
  - Show that two rules provide the keys to understanding.

### Making deformation and strain visible

- For purposes of demonstration, a machine was built to run a latex web at strains of 0.1 inch/inch. Specifications are:
  - Web: 30 mil latex, 5.5 inches wide, 25 Ft long, imprinted with a 0.5 inch reference grid. Modulus, 240 psi.
  - Rollers: Urethane coated, 10 inch face, 3 inch diameter
  - Drive: Pull roller with closed-loop tach feedback -0 to 20 Ft/min
  - Tension: Hysteresis brake 0.5 to 6 Lb
  - Lateral position control: Closed loop at unwind with edge sensor.
  - Unwind and rewind tension: 0.5 Lb using constant torque motors.
  - There is nothing in the machine design that will prevent running materials such as polyethylene and polyester at 1 PLI in the future

### **Test machine schematic**



### **Test machine photo**



### The two rules

- It's customary in engineering to think about stress or force first.
- But, when thinking about lateral web behavior at a roller, it is better to think first about "geometry".
- The two rules that govern web behavior at entry to a roller are basically geometric.
- The rules are:
- The normal entry rule and The normal strain rule.

### The normal entry rule

• A web entering onto a roller will align its direction of travel perpendicular to the roller axis. If it is not initially perpendicular, it will travel laterally on the roller at a rate proportional to the tangent of the angle between the web and the roller until it reaches the perpendicular condition.

There is no evidence that any of the guiding companies in the U.S. knew of this rule prior to 1960. The first published mention of it is in a book by Donald Cambell in 1958. The author independently rediscovered it in 1960 while working at Fife Corporation.

### The normal entry rule for an ideal web



- If the web were a flexible string, it would pivot at the upstream roller and track laterally on the downstream roller until it was perpendicular to it.
- You can easily see this by resting your fingers on a pencil and rolling it across a desktop.

### The normal entry rule for real webs

• The next slides illustrate the beam behavior of a web, first described and analyzed in Shelton's 1968 dissertation.



#### **Initial condition**

- Rollers aligned.
- Tension low (about 0.5 Lb).
- Markings in a square grid pattern were applied to the web when it was relaxed.
- Each vertical line may be thought of as a particle path.



### The roller is pivoted

- The downstream roller is misaligned 5 degrees.
- The web has reached steady state.
- Straight and mutually perpendicular red lines mark:
  - The web centerline if web were not curved
  - The roller axis
- Note that even though the web is curved near the upstream roller, it is perpendicular to the downstream roller axis.



### Effect of increasing tension

- Increasing the tension to 3.7 Lb reduces the curvature.
- The web behaves more like a flexible string.

### **The Normal Entry Rule in Action**

• The following slides show how the normal entry rule causes webs between free-turning uniform idlers to run in a state of pure MD stress.



#### **Initial condition:**

- Brake roller is freewheeling. So, the tension is very low (about 0.5 Lb).
- Enough material has run through the span to reach a steady state.



# Close up of downstream roller:

- Low tension.
- Steady state.
- Vertical red lines mark location of edges.
- The red square provides a reference for strain.



A few seconds after the brake roller is engaged:

- The brake roller has just
  begun to rotate and web
  has reached full tension
  (about 3.7 Lb). The web
  has not had a chance to
  reach steady state and is
  temporarily necked
  down.
- Note that the particle paths at entry to the roller are not perpendicular to the roller axis.



### Many seconds later:

- Steady state at full tension: (uniform MD stress)
- Normal entry rule has caused all particle paths to become perpendicular to the roller axis.
- The web has become elongated in MD direction.
- Poisson ratio causes contraction in CD direction (with zero CD stress).



View of the full span after reaching steady state at full tension (about 3.7 Lb).

- No neck-down at top.
- Neck-down at bottom because of tension drop across the brake roller.
- If the brake roller were an idler, the entire web would be in a state of pure MD stress.

### The normal strain rule

- Based on the conservation of mass mass flow into span must equal mass flow out.
- MD is assumed to refer to the direction normal to the axis of the roller (even if the roller is misaligned)

### **Conservation of mass**



Therefore:  $\Delta x_d / \Delta x_u = V_d / V_u$ 

- A specific patch of web must take the same amount of time to enter a span as it does to leave (in the steady state).
- This must happen regardless of how the shape of the patch changes.

### **Conservation of mass in a span with a concave roller**

- The following slides show how the normal strain rule works at a concave roller.
- For sake of demonstration, the roller profile is unusually large. It is 0.419 inch deep at the midpoint (an arc with a 30 inch radius).
- The edges have an MD velocity 12 % faster than the centerline.

### Conservation of mass in a span with a concave roller (continued)

- The strain profile at the entry to the upstream roller is uniform.
- Mass flow at the concave roller must be the same at every point across the width of the web; because that's what's happening at the upstream roller.
- In the following photos the web is moving very slowly (Approx. 0.01 inch/sec)
- Photos were taken every 5 seconds.



# Deformation at a concave roller

- Edges of the web must move faster than the center.
- Produces higher MD strain at the edges.



### Conservation of mass

- Two horizontal red lines mark location of one of the horizontal grid lines.
- Metal scale provides a fixed reference for locating reference lines.



## Conservation of mass (continued)

- 65 seconds later.
- Next grid line has reached both red marks at the same time.
- Grid lines at the entry to the upstream roller are doing the same thing.

### The implications of conservation of mass

- It looks pretty obvious, doesn't it?
- But, we're not done yet.

### The normal strain rule

- Earlier we said,  $\Delta x_d / \Delta x_u = V_d / V_u$ .
- It can be easily shown that  $\Delta x_d = (1 + \varepsilon_d)$  and  $\Delta x_u = (1 + \varepsilon_u)$ .
- $\varepsilon_u$  is the MD strain at the entry of the upstream roller
- $\varepsilon_d$  is the strain at the entry of the downstream roller
- $V_u$  and  $V_d$  are the respective MD velocities.
- So, at any cross web position at the entry to a roller,  $(1 + \varepsilon_d)/(1 + \varepsilon_u) = V_d/V_u$ . This is the normal strain rule. It is completely general for almost any kind of web and any kind of roller.

*This rule was discovered by the author in Feb. 2003 and first published at IWEB 2005* 

### **Importance of the normal strain rule**

- The MD strain at the entry to a roller can, therefore, be calculated by knowing its shape and the upstream conditions point-by-point across the web.
- Since  $(1 + \varepsilon_d)/(1 + \varepsilon_u) = V_d/V_u$ , the strain at a downstream roller will go up if the velocity goes up.
- The same rule applies when comparing cross web locations on the same roller(assuming uniform upstream conditions).
- This is a big help in explaining things like the spreading of a concave roller.

### **Importance of the normal strain rule (continued)**



- Intuitive insight.
- Rigorous FEA solutions.

### **Spreading with a concave roller**

- Applied wisely, concave rollers are excellent spreaders.
- But, as the next slide will show, the spreading action is limited.
- In this case it's only 0.020 inch at each edge.
- There is no lateral slipping.
- This is in agreement with FEA analysis.



### The spreading action

Vertical red lines were drawn at the edges, near the middle of the span.
They were then copied and moved to the entry of the roller.

### **Transport of strain across a roller**

- The concave roller test provides an excellent opportunity to illustrate how strain is transported across a roller and into the next span.
- With good traction, the strain profile at the entry to the roller will be transported, unchanged, to the exit.
- The next slide illustrates this phenomenon.



The exit side of a concave roller

- The deformation that was created at the entry is carried unchanged across the roller.
- It's deformed even after losing contact with the roller.



# Shape of the relaxed boundary

If you imagine using a flexible scale to draw a straight line across the web at the line of contact and then allowing the web to relax, its apparent that the line will become curved.

### **Shape of the relaxed boundary (continued)**

- Therefore, the shape of the web being analyzed isn't usually rectangular.
- Ordinarily this "defect" in the boundary is negligible.
- But, it can become significant in the case of a short span or nonuniform upstream roller.
- Knowing this can be useful.
- With FEA analysis, it can be used to calculate the effect of strain that is transported into the span.

### **Evolution of shear wrinkles**



- Web before roller is misaligned.
- A reference square has been outlined in red for later comparison.
- Arrow shows direction of travel.

### The shear buckles (troughs)



- Roller pivoted by 7 degrees.
- Web has reached steady state
- Red square from last picture rotated 7 degrees and superimposed.
- Next slide shows area of the square enlarged.

### Shear



- Red square aligned with the bottom reference line normal entry.
- Sides do not align shear strain
- Square has become a parallelogram.
- That's shear.

### The effect of the shear



- Shear causes compressive stress in CD direction,
- Compressive stress causes web to buckle laterally.
- Lengthwise direction of troughs coincides with the direction of maximum MD stress.

#### When troughs become wrinkles



- Web has to bend to conform to the roller.
- Radial forces develop that push the web and the trough toward the roller surface.

### When troughs become wrinkles (continued)



- Radial force tries to squeeze trough flat by compressing or displacing adjacent material.
- If the sides of trough can't support the radial force, it collapses into a wrinkle.

### **Comparison of the shear troughs at different times.** They're moving from bottom to top.



They move for the same reason that a web moves laterally at a misaligned roller. But, since they aren't anchored at the upstream roller they don't pivot. They just travel laterally without changing their angle.

Relative to the web, the troughs don't move in the CD direction. They only move in the MD direction. In the pictures below, a red cross marks the same web location at different times.



