# Effects of Concave Rollers, Curved-axis Rollers and Web Camber on the Deformation and Translation of a Moving Web

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J. L. Brown

Essex Systems

# Assumptions

- Traction on rollers
- Negligible turning torque for rollers
- No viscolelastic or inertial effects

# Earlier Work

- Shelton's example for beam modeling – Dissertation 1968.
- Swanson's spreading experiment – IWEB 1997.
- Swanson's cambered web experiments – IWEB 1999.
- Shelton's survey of cambered web problems – IWEB 1997
- Markum and Good's concaver roller experiments – IWEB 2001
- Olsen's proposal to use of frozen-in strain concept for nonuniform webs – IWEB 2001

# Particle Displacements

Particle displacement in $x$ direction = $u$

Particle displacement in $y$ direction = $v$

# Plane Stress, *x* & *y* Strains

- Longitudinal strain

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

- Cross web strain

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

# Plane Stress, Angular Displacements

- Angular displacement of infinitesimal element originally parallel to x-axis

$$v_x = \frac{\partial v}{\partial x}$$

- Angular displacement of infinitesimal element originally parallel to y-axis

$$u_y = \frac{\partial u}{\partial y}$$

# Plane Stress, Shear & Rotation

- Shear

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

- Rotation

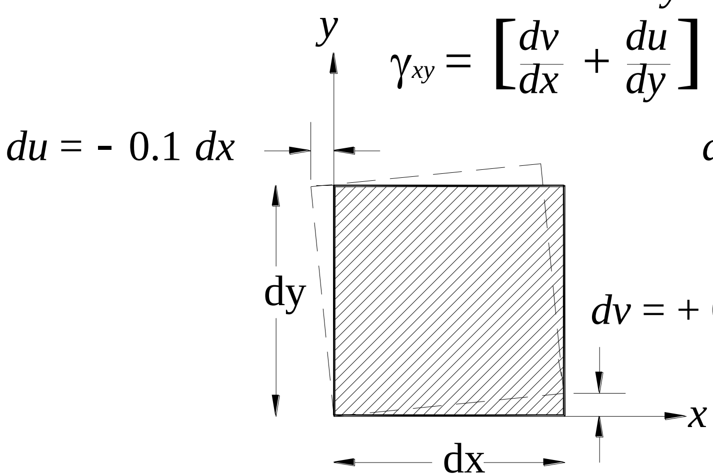
$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

# Interpretation of Rotation

$$\frac{du}{dy} = -0.1 \qquad \frac{dv}{dx} = +0.1$$

$$\omega_z = .5\left[\frac{dv}{dx} - \frac{du}{dy}\right] = 0.1$$

$$\gamma_{xy} = \left[\frac{dv}{dx} + \frac{du}{dy}\right] = 0$$

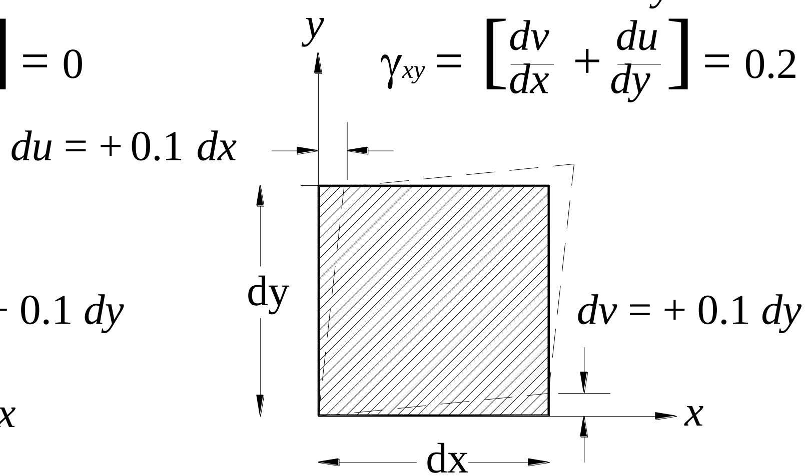



Rotation without shear

$$\frac{du}{dy} = +0.1 \qquad \frac{dv}{dx} = +0.1$$

$$\omega_z = .5\left[\frac{dv}{dx} - \frac{du}{dy}\right] = 0$$

$$\gamma_{xy} = \left[\frac{dv}{dx} + \frac{du}{dy}\right] = 0.2$$




Shear without rotation

# Deformed Coordinates

- Deformed *x* coordinate

$$\xi = x + u$$

- Deformed y coordinate

$$\eta = y + v$$

# Stress

- x-axis stress

$$\sigma_x = \frac{E}{1-\mu^2}\left[\varepsilon_x + \mu\left(\varepsilon_y\right)\right]$$

- y-axis stress

$$\sigma_y = \frac{E}{1-\mu^2}\left[\varepsilon_y + \mu\varepsilon_x\right]$$

- Shear

$$\tau_{xy} = \frac{E}{2(1+\mu)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]$$

# Novoshilov's Equations of Equilibrium

- X direction

$$\frac{\partial}{\partial x}\left[\sigma_x - \omega_z \tau_{xy}\right] + \frac{\partial}{\partial y}\left[\tau_{xy} - \omega_z \sigma_y\right] = 0$$

- Y direction

$$\frac{\partial}{\partial y}\left[\sigma_y + \omega_z \tau_{xy}\right] + \frac{\partial}{\partial x}\left[\tau_{xy} + \omega_z \sigma_x\right] = 0$$

# Downstream Boundary Conditions

- Normal Entry Rule
- General case

$$\psi = \tan^{-1}\left[v_x dx + \left(1+\varepsilon_y\right)dy\right]\left[\left(1+\varepsilon_x\right)dx + u_y dy\right]^{-1} = \theta_r$$

- For uniform web $\tan^{-1} v_x (1+\varepsilon_x)^{-1} \approx v_x = \theta_r$

$\psi$ = angle of tangent to particle path, relative to x-axis.

$\theta_r$ = angle of roller axis relative to y-axis

# Downstream Boundary Conditions

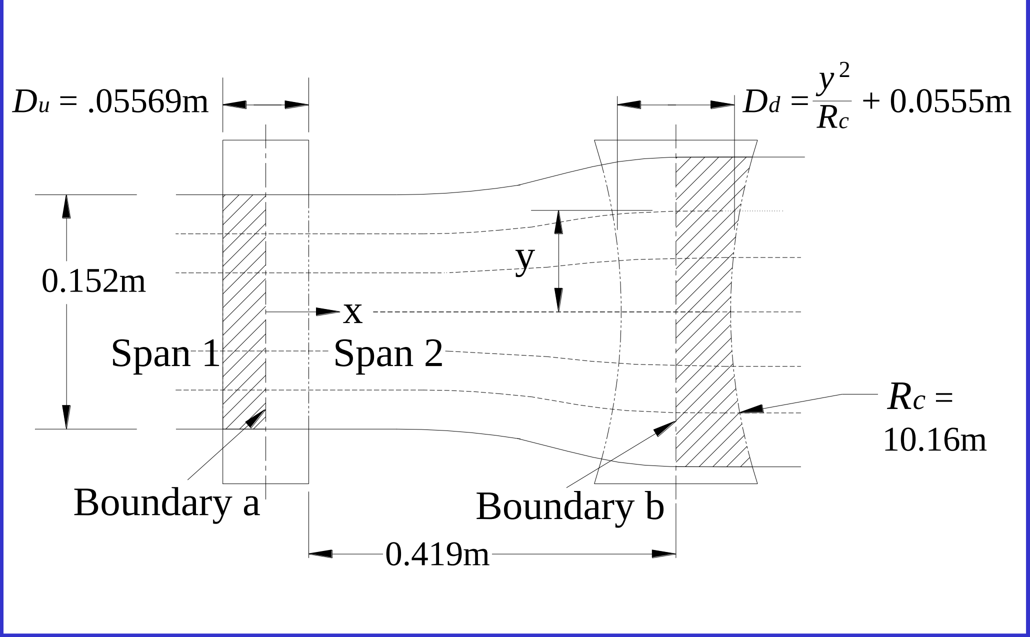
- Normal Strain Rule (approximation)

$$\varepsilon_x = 1 - \frac{V_u}{V_d}\left(1 - \varepsilon_o\right)$$

$V_u$ and $V_d$ are upstream and downstream circumferential roller velocities.

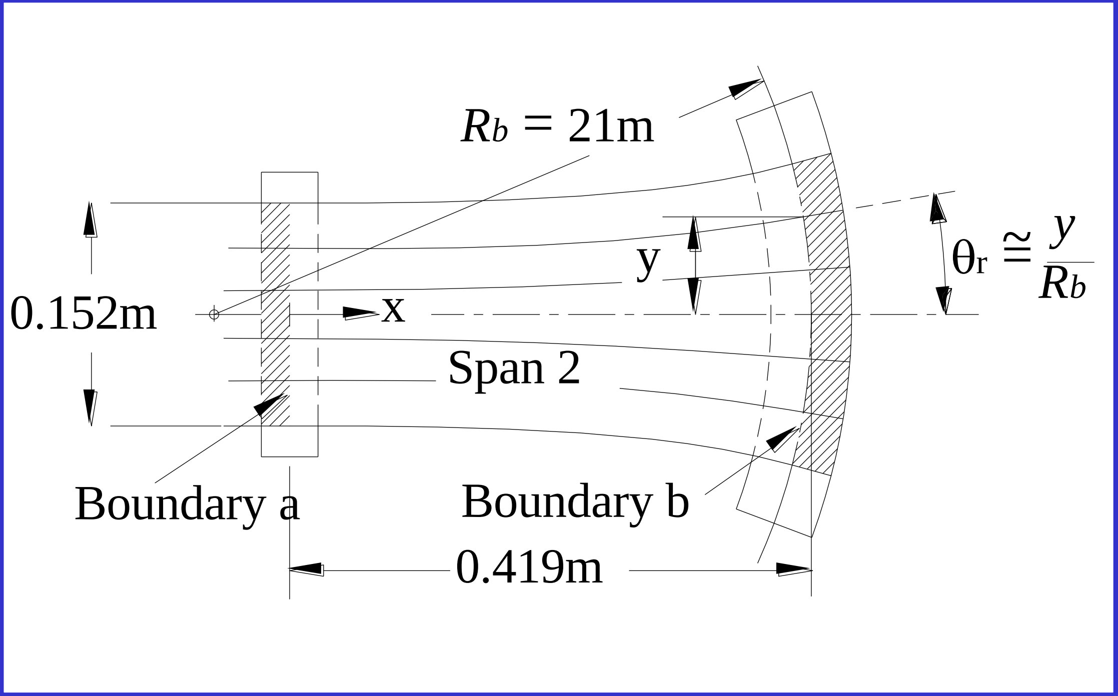
$\varepsilon_o$ is the longitudinal strain at the entry to the upstream roller.

# Comparison of Concave & Curved-axis Rollers




Concave roller based on Markum and Good experiment.

# Comparison of Concave & Curved-axis Rollers




Curved-axis roller using same span parameters as Markum and Good experiment.

# Comparison of Concave and Curved-axis Spreader Rollers

- Span parameters

| Material | Modulus (Mpa) | Caliper (mm) | Width (m) | Length (m) | Tension (N) |
|---|---|---|---|---|---|
| LDPE | 165.5 | 25.4 | 0.152 | 0.419 | 17.8 |

- Concave roller
  - Profile radius $R_o$ = 10.16 m
  - Center diameter, $D_c$ = 55.5 mm

$$D_d(y) = 2\left(\frac{D_c}{2} + \frac{y^2}{2R_o}\right) == 2\left(0.02776 + 0.0492y^2\right)$$

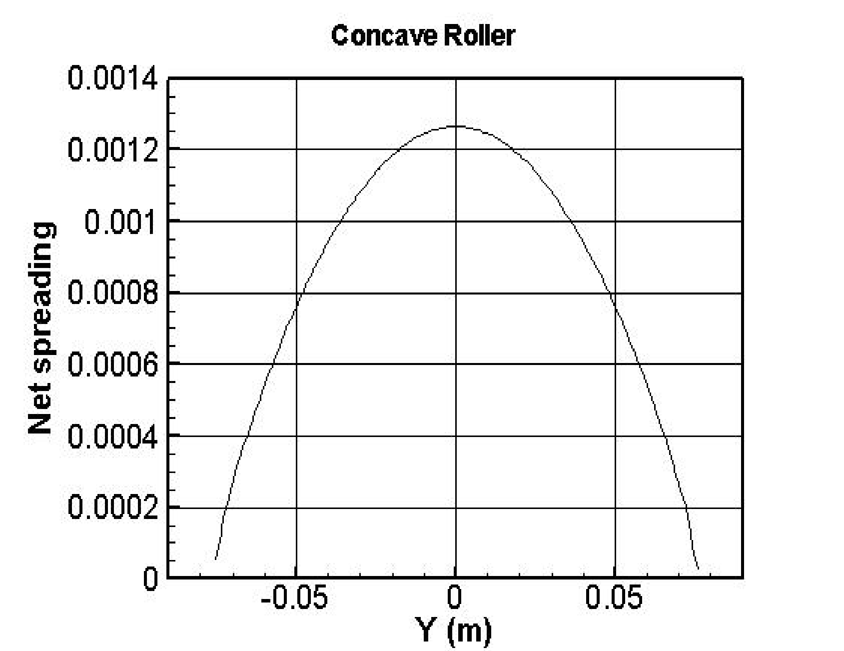
- Curved-axis roller
  - Radius = 21 m

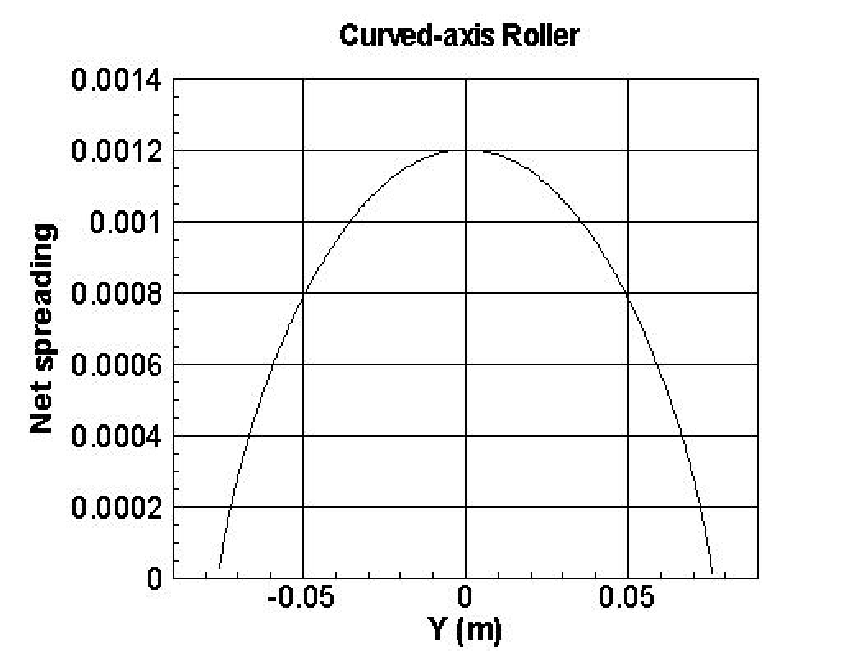
# Spreader Boundary Conditions

- Sides $\sigma_n = 0$ $\tau_n = 0$
- Upstream roller $u = 0$ $v = -\mu\varepsilon_o y$
- Downstream roller

| Concave | Curved-axis |
|---|---|
| $\varepsilon_x = 1 - \frac{V_u(1-\varepsilon_o)}{V_u\left[1+f(y)\right]} \cong \varepsilon_o + f(y)$ | $\varepsilon_x = 1 - \frac{V_u}{V_d}(1-\varepsilon_o)$ |
| $f(y) = (1.767m^{-1})y^2 - .00340$ | $v_x = \theta_r = \sin^{-1}\left(\frac{y}{R_b}\right) \cong \frac{y}{R_b}$ |
| $v_x = 0$ | |

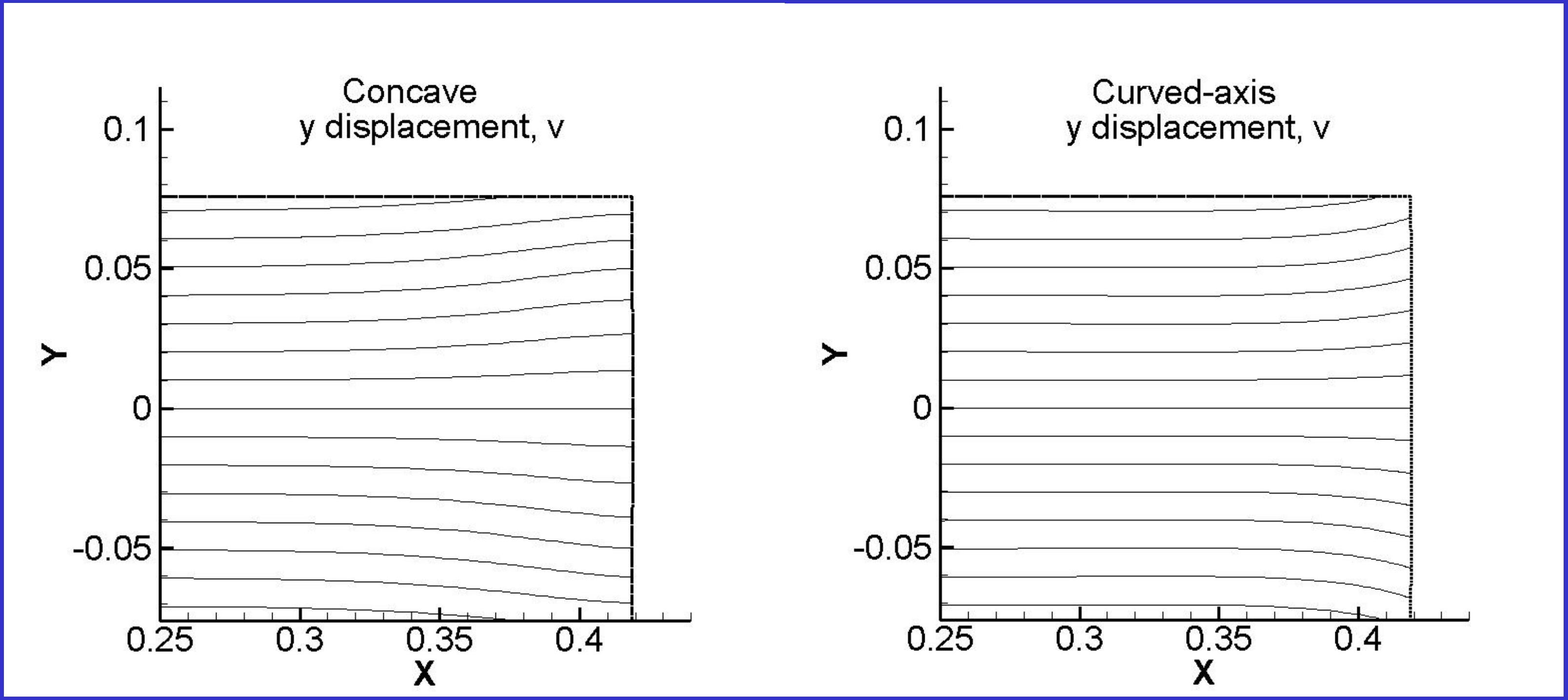
# Net Spreading



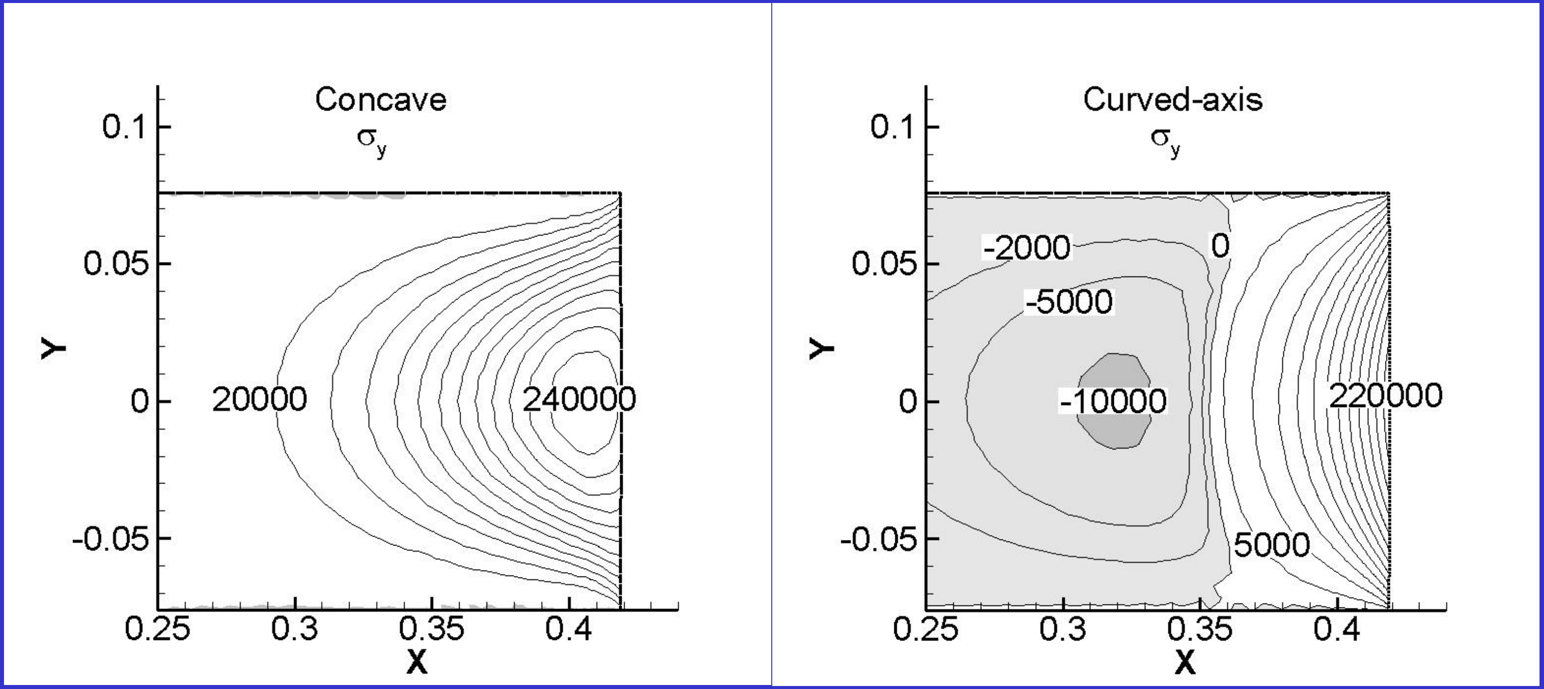

Net spreading strain = $\varepsilon_s = \varepsilon_y + \mu\varepsilon_x$

# *v* Displacement Contours




These contours of constant *v* are not particle paths. But, they have similar slopes

# $\sigma_y$ Contours




The rollers show similar patterns of cross web stress at the downstream roller. The curved-axis spreader shows a region of compressive stress (shaded) . But, levels are low. There is a similar pattern for the concave roller that is just beyond the area shown in the graph. It has a peak value of –825 Pa.

# Magnification of Lateral Errors by Concave Rollers

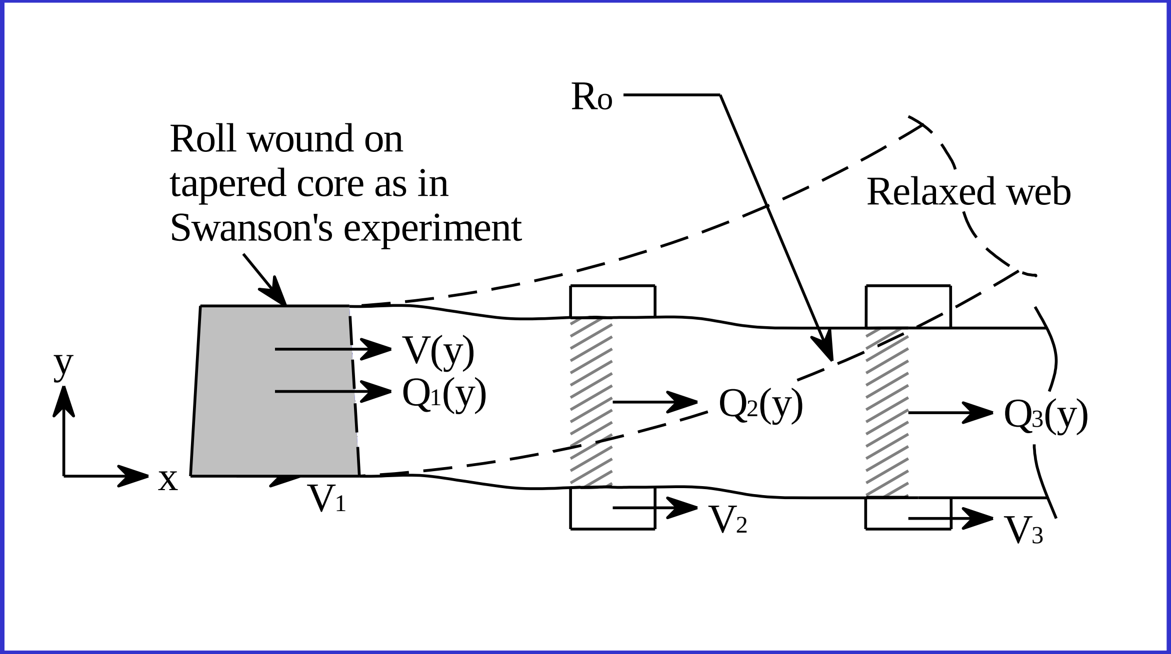
- There has been a presumption that because of
  - The lateral shifts seen with tapered rollers.
  - And the centering behavior of a crowned roller.

  concave rollers would amplify lateral registration errors.
- This is technically true. However, the effects are generally small. For example, with the extreme profile of "parabolic 2" used in the Markum and Good experiment, the magnification would be only 1.12 (1 inch error at upstream roller increases to 1.12 inch error at spreader). This ratio held for lateral errors up to ½ web width. Doubling the profile radius reduced the magnification to 1.06.

# Magnification of Errors by Curved-axis Rollers

- Curved-axis rollers have never been suspected of amplifying lateral errors. Analysis shows they do. But, the effect is much smaller than for concave rollers of the same spreading capability.
- The curved-axis roller in the previous example amplifies errors by 1.015.
- Although the curved-axis error magnification is less, the net difference in error at the spreader is only 10%.
- So, when the maintenance problems of curved-axis rollers are considered, the concave roller may often be a better choice. Particularly if it is designed to have no more spreading than absolutely necessary.

# The Cambered Web




Start by assuming that a cambered web is created on a tapered core in the manner described by Swanson in his 1999 IWEB paper.

# Mass Flow in the Cambered Web

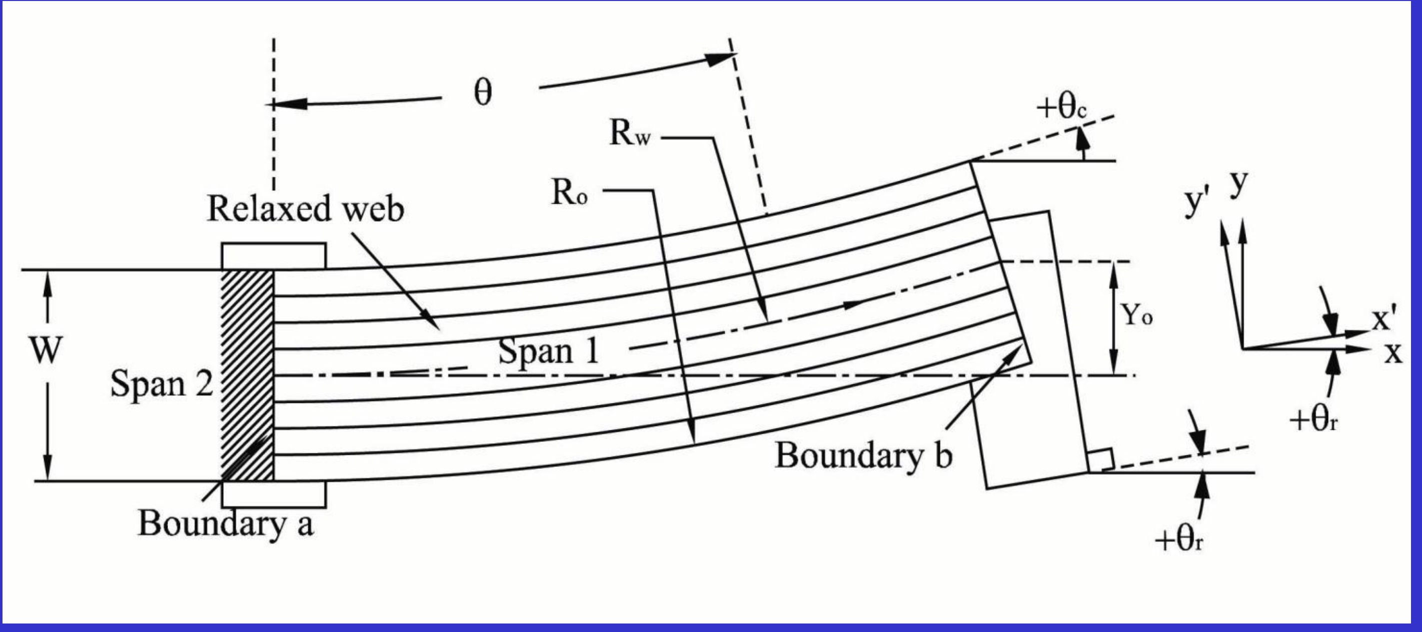
Applying the mass flow concepts of the normal strain rule to the cambered web leads to the following relationship for the longitudinal strain at the entry to a roller.

$$\varepsilon_x = 1 - \frac{V_u}{V_d}\frac{R_o - y}{R_o}\left(1 - \varepsilon_{xo}\right)$$

- $V_u$ and $V_d$ are the upstream and downstream roller velocities.
- $R_o$ is the radius of curvature of the outside edge of the web.
- $\varepsilon_{xo}$ is the longitudinal strain at the outer edge of the upstream span.
- And $\varepsilon_{xo}$ is zero immediately following the unwinding roll.

# Normal Entry for a Cambered Web

For the P. D. E. model, the boundaries are those of the relaxed web. So, the particle paths of the relaxed web are circular arcs.



# Normal Entry for a Cambered Web

The curved reference geometry alters the definition of $\psi$, the tangent angle to the deformed particle paths.

$$dy = -x/\left(r^2 - x^2\right)^{1/2} dx = \tan\theta \, dx$$

So, the tangent angle to the deformed particle path at the roller is,

$$\psi = \tan^{-1}\left\{\left[v_x + \left(1+\varepsilon_y\right)\tan\theta_c\right]\left[\left(1+\varepsilon_x\right)+u_y \tan\theta_c\right]^{-1}\right\} = \theta_r$$

And,

$$v_x \cong \theta_r - \theta_c$$

where $v_x$ is the angular displacement in the $y$ position of a web particle relative to $x$, $\theta_c$ is the angle of camber at the downstream roller and $\theta_r$ is the roller misalignment.

# Boundary Conditions at the Edges

The normal stress and the tangential shear will be zero at the edges.

$$\sigma_n = 0 = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha$$

$$\tau_n = 0 = \left(\sigma_y - \sigma_x\right)\sin\alpha\cos\alpha + \tau_{xy}\left(\cos^2\alpha - \sin^2\alpha\right)$$

$\alpha$ is the angle of the boundary relative to the x-axis.

# Boundary Conditions Upstream

The *x* displacement, *u* , is set to zero. $u = 0$

The *y* displacement, *v* , can be determined by a solution for the upstream span. If that is not available, the longitudinal strain for the downstream roller can be used to estimate *v*.

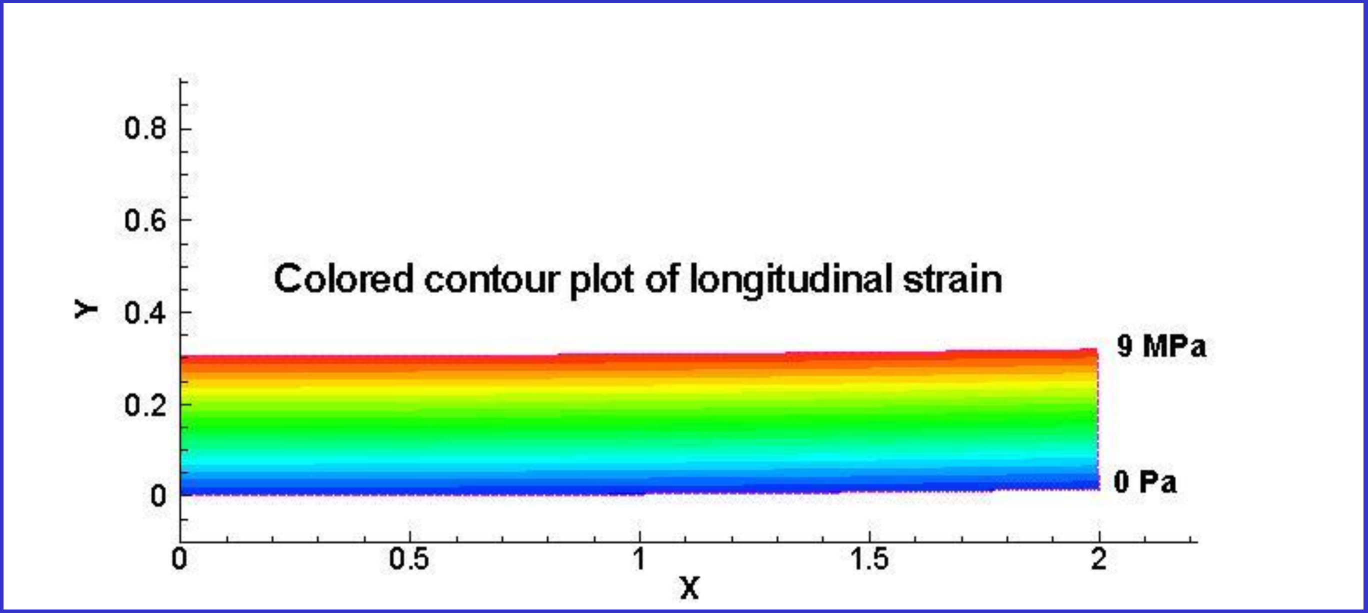
$$v = -\mu \int \varepsilon_x dy + C$$

C can be chosen arbitrarily to position the web vertically.

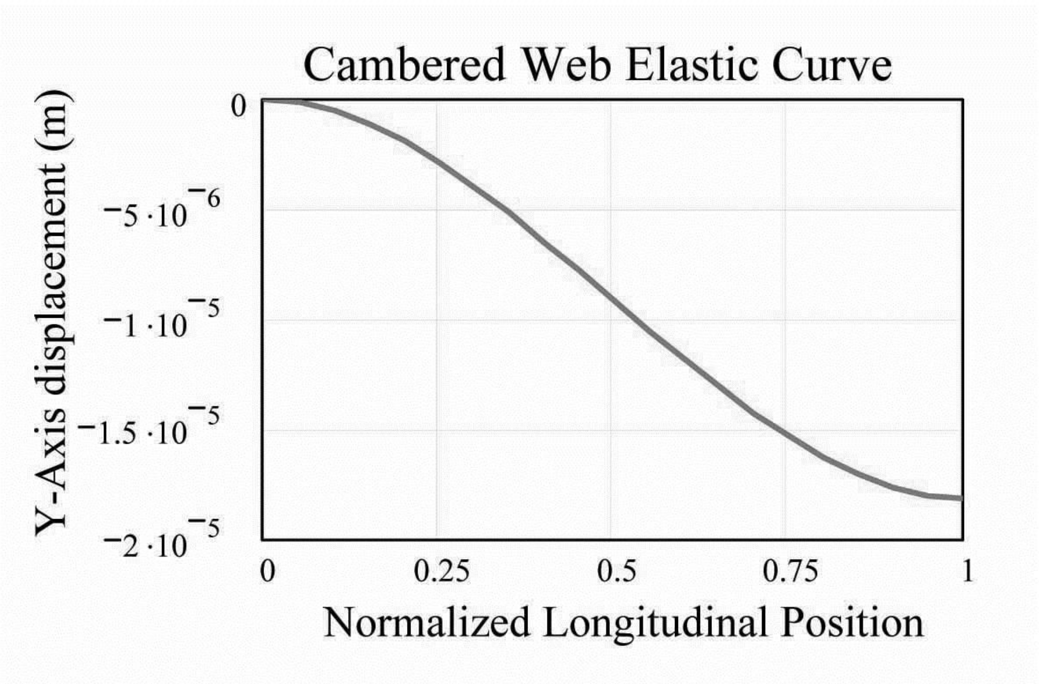
# Comparison of P. D. E. Results With One of Swanson's 1999 Tests, $\sigma_x$

$L$ = 2m, $W$ = 0.3048m (12 In.), $E$ = 4.14e9Pa, $h$ = 23.4µm, $T_{avg}$ = 32.5N, $R_w$ = 139m. (Neutral axis of relaxed web is 14.4mm above y-axis at x = 2m)

Longitudinal stress for aligned rollers, shown on undeformed domain. Note the surprising uniformity from end to end.



# Elastic Curve of Neutral Axis




Plotted relative to a horizontal line through the center of the web at the upstream roller.

So, deflection at the downstream roller is -18 μm (below the position of a perfectly straightened web).

# Result of Comparison

- Swanson measured –300µm deflection. The model predicted –18.1µm. Both of these values are very small compared to the initial camber offset of 14,400µm in the relaxed web (0.125 and 2.1 %). The difference could be due to measurement error or some discrepancy in the test setup.
- Both the model and the test showed that a cambered web becomes nearly straight between aligned rollers

# Conclusions From Cambered Web Modeling With Parallel Rollers

- The curvature at the downstream roller is, as Swanson predicted, between 0 and $1/R_w$ (actually a very small positive value).
- A cambered web has a small negative displacement relative to a perfectly straight web. This is in agreement with Swanson's tests. It was true in all 9 of his reported results.
- The longitudinal stress will increase linearly from a minimum at the convex edge to a maximum at the concave edge.
- The longitudinal tension profile across the web will be very uniform throughout the span.

# Beam Model for a Cambered Web

- A beam model was developed using the normal strain relation at the downstream roller to estimate the fourth boundary condition based on the assumption that

$$\sigma_x \cong E\varepsilon_x$$

- Thus if, $$\varepsilon_x = 1 - \frac{V_u}{V_d}\frac{R_o - y}{R_o}\left(1 - \varepsilon_{xo}\right)$$

- The end moment is $$M_L = Eh\int_{-W/2}^{W/2} \frac{R_o}{R_o + W/2}\frac{V_u}{V_d}\left(1 - \varepsilon_o\right) y\,dy$$

# Beam Model for a Cambered Web

- The P. D. E. model shows this moment estimate to be very accurate for a wide range of L/W.
- The resulting model is an equation that has the same algebraic form as Shelton's misaligned roller equation.

$$y_c = C_1 \sinh\left(K_c x\right) + C_2 \cosh\left(K_c x\right) + C_3 x + C_4$$

- But, $K_c$ and the coefficients are different.

# Beam Model for a Cambered Web

- $K_c$ has an additional factor to account more completely for the effect of tension on curvature.

$$K_c^{\,2} = \frac{T}{EI\left(1-\frac{T}{AE}\right)\left(1+\frac{nT}{AG}\right)}$$

- And the curvature at the downstream roller is the difference between the initial curvature and the effect of the end moment.

$$\frac{1}{\rho_L} = \frac{1}{R_w} - \frac{M_L}{EI}\left(1-\frac{T_{avg}}{AE}\right)^{-1}\left(1+\frac{nT_{avg}}{AG}\right)^{-1}$$

# Beam Model for a Cambered Web

- The coefficients reveal that the solution is the sum of the usual misaligned roller equation plus new terms that add the effect of the camber and end moment.

$$C_1 = \left[ -\frac{\theta_r}{K_c}\cosh\left(K_c L\right) + \frac{\phi}{R_w K_c^{\,2}}\sinh\left(K_c L\right) \right] \left[ \cosh\left(K_c L\right)\left(1+\frac{nT_{avg}}{AG}\right) - 1 \right]^{-1}$$

$$C_2 = \left[ \frac{\theta_r}{K_c}\sinh\left(K_c L\right) + \frac{\phi}{R_w K_c^{\,2}}\left(1+\frac{nT_{avg}}{AG} - \cosh\left(K_c L\right)\right) \right] \left[ \cosh\left(K_c L\right)\left(1+\frac{nT_{avg}}{AG}\right) - 1 \right]^{-1}$$

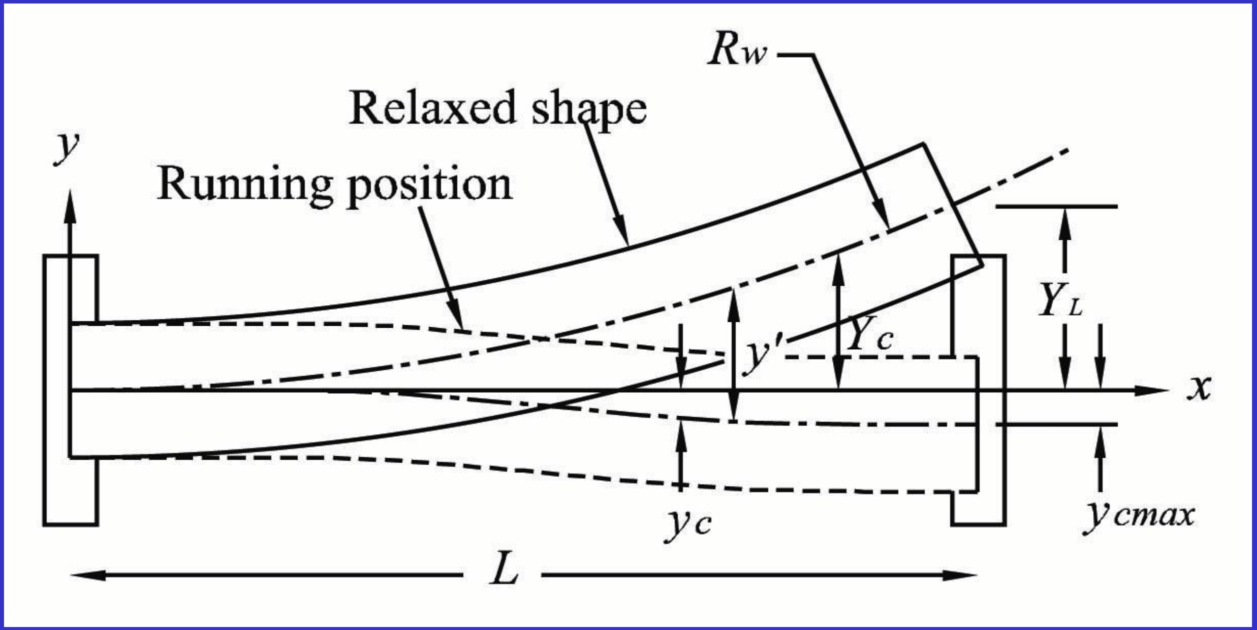
$$C_3 = -C_1 K_c \left(1+\frac{nT_{avg}}{AG}\right)$$

$$C_4 = -C_2$$

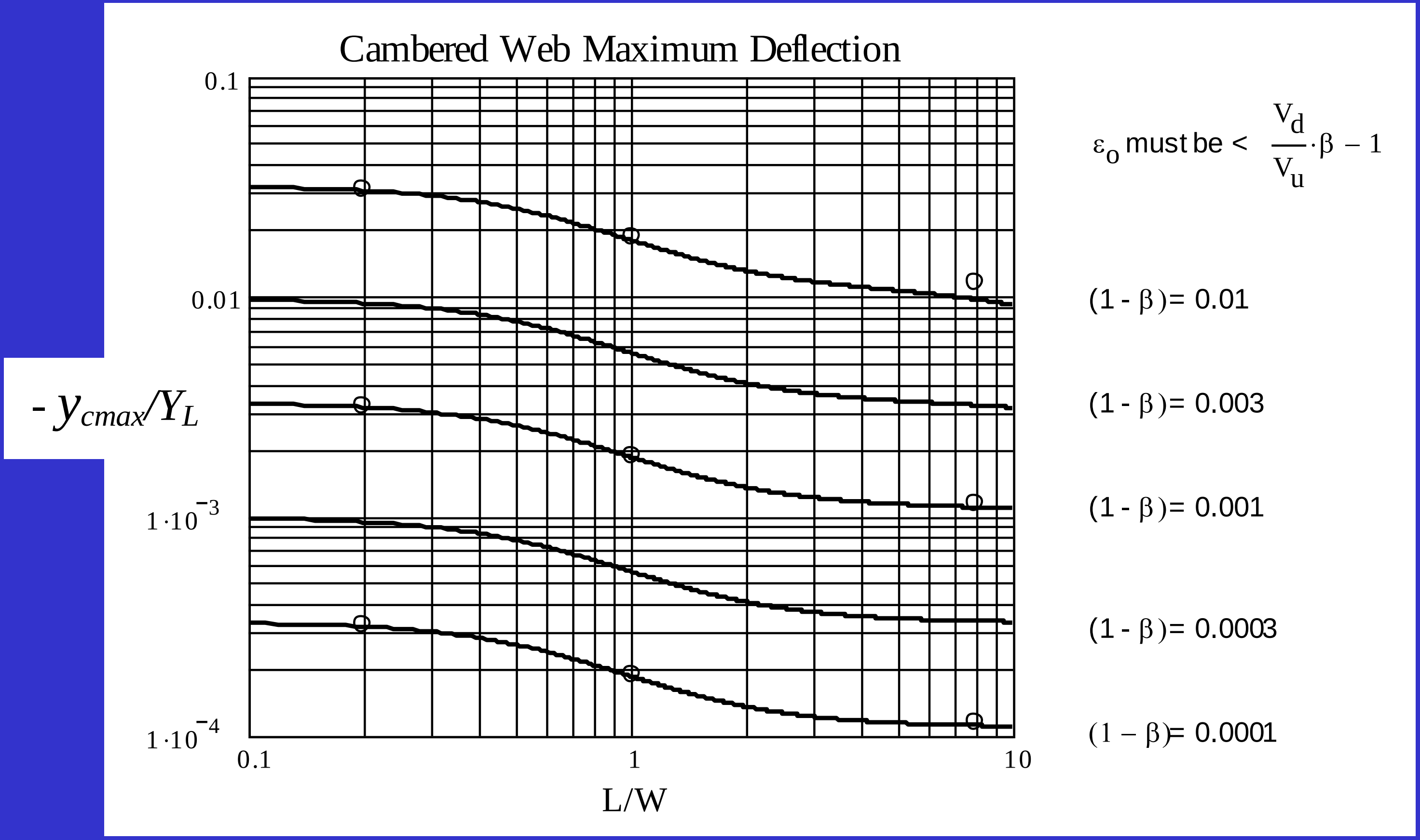
$\phi$ is a term related to the end moment.

# Comparison of the P. D. E. and Beam Models for a Cambered Web

- In the next slide the value of $y_{cmax}/Y_L$ will be plotted against L/W.



# Comparison of the P. D. E. and Beam Cambered Web Models



# Conclusions

- It has been shown that the new method can be successfully applied to the following situations.
  - The spreading behavior of concave and curved-axis rollers.
  - The deflection and deformation of a cambered web.
  - The development of a beam theory model for a cambered web.
- The new method can evaluate the potential for damage to webs by producing precise and detailed descriptions of stress/strain fields throughout spans.
- It is evident that much more can be done in exploring these and other applications. Additional simplified models can be developed and where that is not possible, results can be tabulated for everyday use.

# Conclusions

Beyond illustrating the capabilities of the method, the following things have been accomplished.

- A beam model of the cambered web has been developed and shown to produce the same results as the new method for small strains.
- It has been shown that the concave and curved-axis rollers behave very much alike and that concave rollers have an undeserved bad reputation.
- It has been shown that camber in a web can produce large variation in longitudinal stress across the width. But, it does not cause large lateral misalignment errors.