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Alternative Derivation of Normal Strain Rule and an Unnecessary Approximation

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In the diagram below, the web is assumed to be running in a steady state with good traction on the rollers. At the entry to the rollers, the normal entry rule requires that the particle paths be normal to the axis of rotation of a roller surface. The area marked (a) represents an infinitesimal area with sides parallel to two particle paths. The area marked (a') is the same portion of web at the moment that it enters onto the downstream roller. The roller surface velocities are V_u and V_d. Δx is the relaxed length of the area (a) in direction x. ε_{xu} is the strain normal to the upstream roller. ε_{xd} is the strain normal to the downstream roller. Conservation of mass requires that the mass flow into and out of the span for any portion of web between two particle paths be constant. Otherwise material would accumulate in the span and a steady state would not exist. For this to be true, each area must travel past the line of roller contact in the same length of time. Therefore,

$$\Delta x\left(1+\varepsilon_{xu}\right)=V_u\,\Delta t \qquad (1)$$

and

$$\Delta x\left(1+\varepsilon_{xd}\right)=V_d\,\Delta t \qquad (2)$$

Equating (1) and (2) and solving for ε_{xd}

$$\varepsilon_{xd}=\frac{V_d}{V_u}\left(1+\varepsilon_{xu}\right)-1 \qquad (3)$$

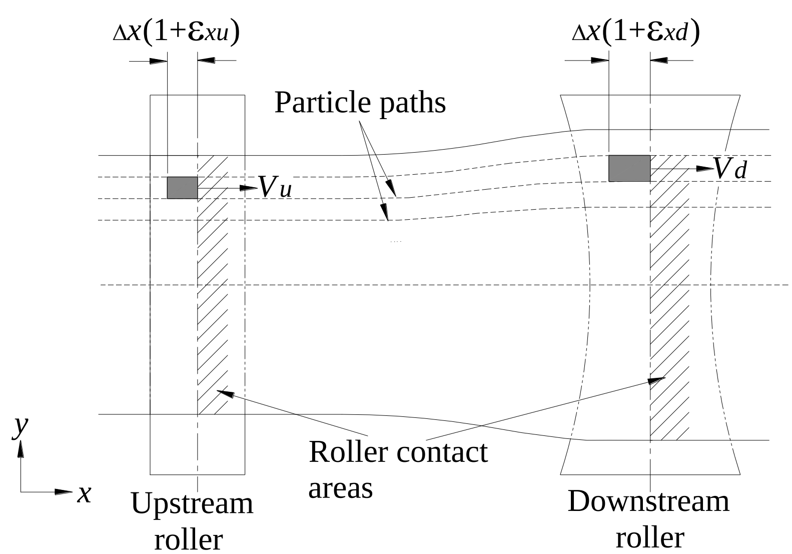

Figure 1

Equation (3) is not the same as that originally developed [1] in "A New Method for Analyzing the Deformation and Lateral Translation of a Moving Web", presented at IWEB 2005. However, the two equations are numerically equivalent. In the method of derivation used in the IWEB paper, separate expressions were developed for the mass flows at the upstream and downstream rollers. For example, the mass flow upstream is,

$$Q_i = \frac{V_u(dy)(h)(\rho_o)}{(1+\varepsilon_{xu})} \tag{4}$$

where dy is the increment of width, h is the thickness and ρ_o is thickness. The strain in the denominator looked like a potential nonlinearity. So, the approximation,

$$(1+\varepsilon_{xu})^{-1} \approx (1-\varepsilon_{xu}) \tag{5}$$

was used. This led to the relationship,

$$\varepsilon_{xd} \approx 1 - \frac{V_u}{V_d}(1-\varepsilon_{xu}). \tag{6}$$

Although (6) is algebraically different than (3), the numerical difference in ε_{xd} is insignificant so long as ε_{xu} is very small and V_u/V_d is close to 1. Both of these requirements are met in any practical web handling problem. Note that when $V_u/V_d = 1$ they are exactly equivalent and that was the value assumed for all the examples in the paper. Nevertheless, the concern about a nonlinearity was misguided. Leaving (4) alone, along with its companion expression downstream, would have led to (3) which is exact and linear.

See the note below for additional details.

Comparison of equations (3) and (6)

Upstream strain, exu	Vd/Vu	1-(Vu/Vd)(1-eo)	(Vd/Vu)(1+eo)-1	% error
0	1	0.000000	0.000000	0
0.001	1	0.001000	0.001000	0
0.005	1	0.005000	0.005000	0
0.01	1	0.010000	0.010000	0
0	1.001	0.000999	0.001000	-0.0999001
0.001	1.001	0.001998	0.002001	-0.149825187
0.005	1.001	0.005994	0.006005	-0.183080866
0.01	1.001	0.010989	0.011010	-0.190635885
0	1.01	0.009901	0.010000	-0.99009901
0.001	1.01	0.010891	0.011010	-1.080026259
0.005	1.01	0.014851	0.015050	-1.319035558
0.01	1.01	0.019802	0.020100	-1.482685582

More about the shape of area (a')

[1] The approximation is used in "Effects of Concave Rollers, Curved-Axis Rollers and Web Camber on the Deformation and Translation of a Moving Web."

Area (a') shown in Figure 1 will not necessarily be rectangular. In cases, such as concave rollers, there will be shear stress at some points across the web at the roller entry. This does not, however, affect the validity of the normal strain rule. As illustrated in Figure 2, the shear strain will be equal to $\phi' - \phi$. Since ϕ' is the normal entry angle, it will be zero. Therefore, (a') becomes a parallelogram with an area equal to a rectangle of the same height and length and the rate of material transport past the line of contact at the roller will be unchanged.

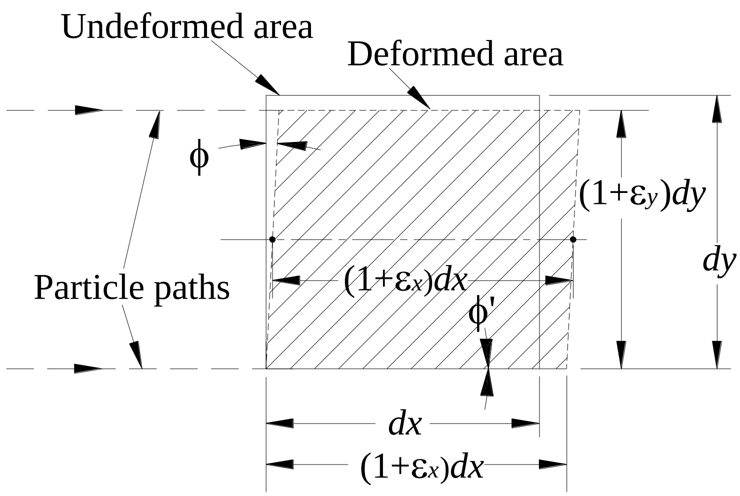

Figure 2