Two-Dimensional Behavior of a Thin Web on a Roller

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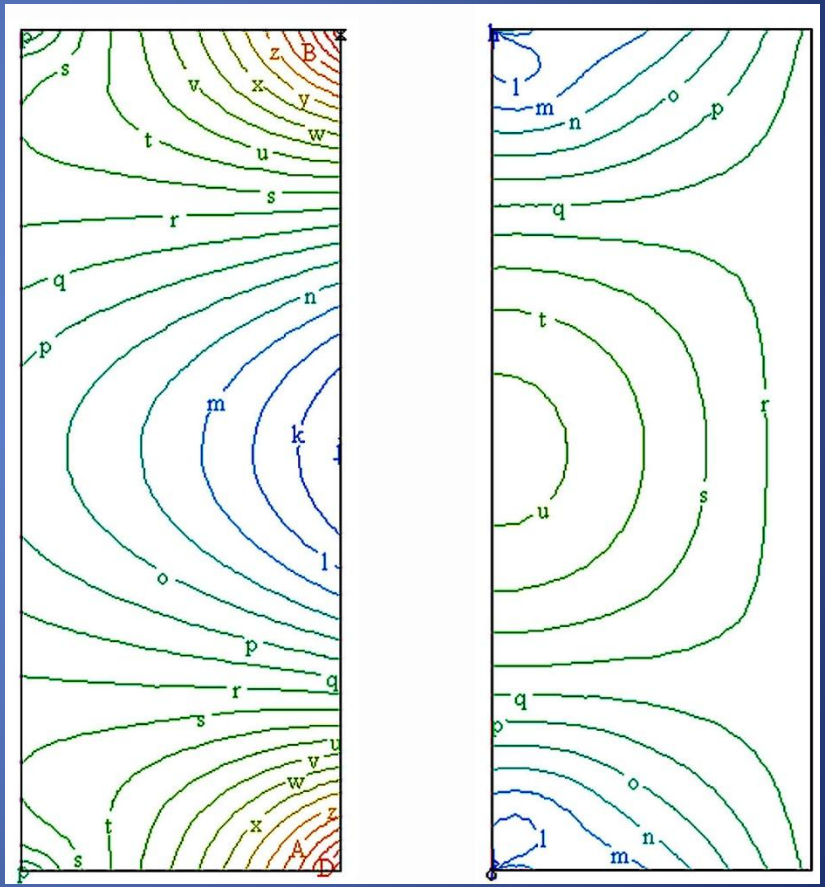
Traction is a two-dimensional phenomenon

- Everyone knows that the idealized rectangular patches that we use to model stick (entry) and microslip (exit) zones are not realistic. But, we’ve had nothing better.
- The circumferential extent of one zone relative to the other varies across the web, except in the most ideal case of pure MD stress at an aligned and uniform roller.
- It affects the amount of spreading that can be supported on concave and curved rollers.
- The absence of a stick zone over even a small portion of the web width will negate the most fundamental assumption we make in the analysis of lateral behavior – isolation of spans.

For example

- This is a plot of the MD stress contours of the entering and exit spans of a concave roller.
- To get the boundary conditions at the upstream end of the exit span, it was assumed
  - That there was no microslip zone on the roller.
  - The shape of the relaxed web edge exiting the roller was forced to match the relaxed shape at the entry.
- Clearly something is missing and it happened on the roller.



This paper is a beginning

It provides a criterion for predicting when a web will lose traction at the entry of a roller and, starting from first principles, defines some of the concepts and issues necessary for dealing with the microslip zone.

Method of analysis

- This analysis is based on the method presented in the author's 2005 paper "A New Method for Analyzing the Deformation and Lateral Translation of a Moving Web".
- The web span is assumed to be in a steady state so that it can be treated as a stationary membrane with boundary conditions that are consistent with the flow of an elastic medium. At the downstream roller:
  - Particle motion on entry to the downstream roller must match the velocity vector of the surface of the roller (normal entry rule).
  - Mass flow at each point across the web must remain constant from one end of the span to the other (normal strain rule).

Method of analysis

- For a 2-D problem, the essential physics of the web is embodied in two 2nd order partial differential equations.
  - These are the classical equations of elasticity – the equilibrium of forces – and the constitutive equations specifying the relation between stress and strain in the material.
  - If finite displacements are involved, these equations may require nonlinear terms to describe the deformation.
- Appropriate boundary conditions are established and the equations are solved numerically.
  - One of the easiest ways to do this today is to use a general-purpose FEA PDE solver.
  - If the solver converges and the boundary conditions are met, then the solution satisfies the physical laws inherent in the equations.

Prior work

- An excellent review of traction in web handling was presented by Dilwyn Jones at the 2001 IWEB conference. It has an excellent bibliography.
- Among the papers cited is the excellent work done at the WHRC by Dr. Good and Keith Ducotey.
- In 1995 Jones and Zahlan reported on an interesting qualitative study using continuum mechanics software (ABAQUS).
- The one-dimensional capstan model continues to be the mainstay of web handling.

Assumptions

- The effect of air lubrication is accounted for in the coefficient of friction.
- The web is in a steady state.
- In the stick zone, the static coefficient of friction is used. The dynamic value applies in the microslip zone.
- In this analysis the linear equations of elasticity will be used when the web is on the roller. However, there is no obstacle to using the nonlinear equations if needed.
- The roller or web may be nonuniform. But the roller has a straight axis.
- The x-y coordinate system will be rotated to match the roller position.

Boundary conditions

- At the upstream end of a span the MD displacement is zero and the CD displacement is $-\mu\sigma_x/E$ (Poisson contraction).
- At free edges, CD and shear stresses are zero.
- At the downstream end, particle paths are aligned with the direction of motion of the roller surface (normal entry). Strain in the direction of particle paths is related to the strain at the entry of the previous roller by the equations below (normal strain).

$$\frac{\partial v}{\partial x} = (1+\varepsilon_{\tilde{x}})\tan(\theta_r) \qquad \frac{1+\varepsilon_{\tilde{x}}}{1+\varepsilon_o} = \frac{V_d}{V_o}$$

Generalized conditions for steady state flow of an elastic solid

- The boundary conditions of the previous slide are actually special cases of general relationships that apply to steady state flow of an elastic solid. In what follows, it will be useful to have these concepts defined.
- In the 2005 paper it was shown that the steady state trajectory of a particle in a deformed web makes an angle ψ with the MD direction defined by,

$$\psi = \tan^{-1}\left(\frac{\partial v}{\partial x}\frac{1}{1+\varepsilon_{\psi}}\right)$$

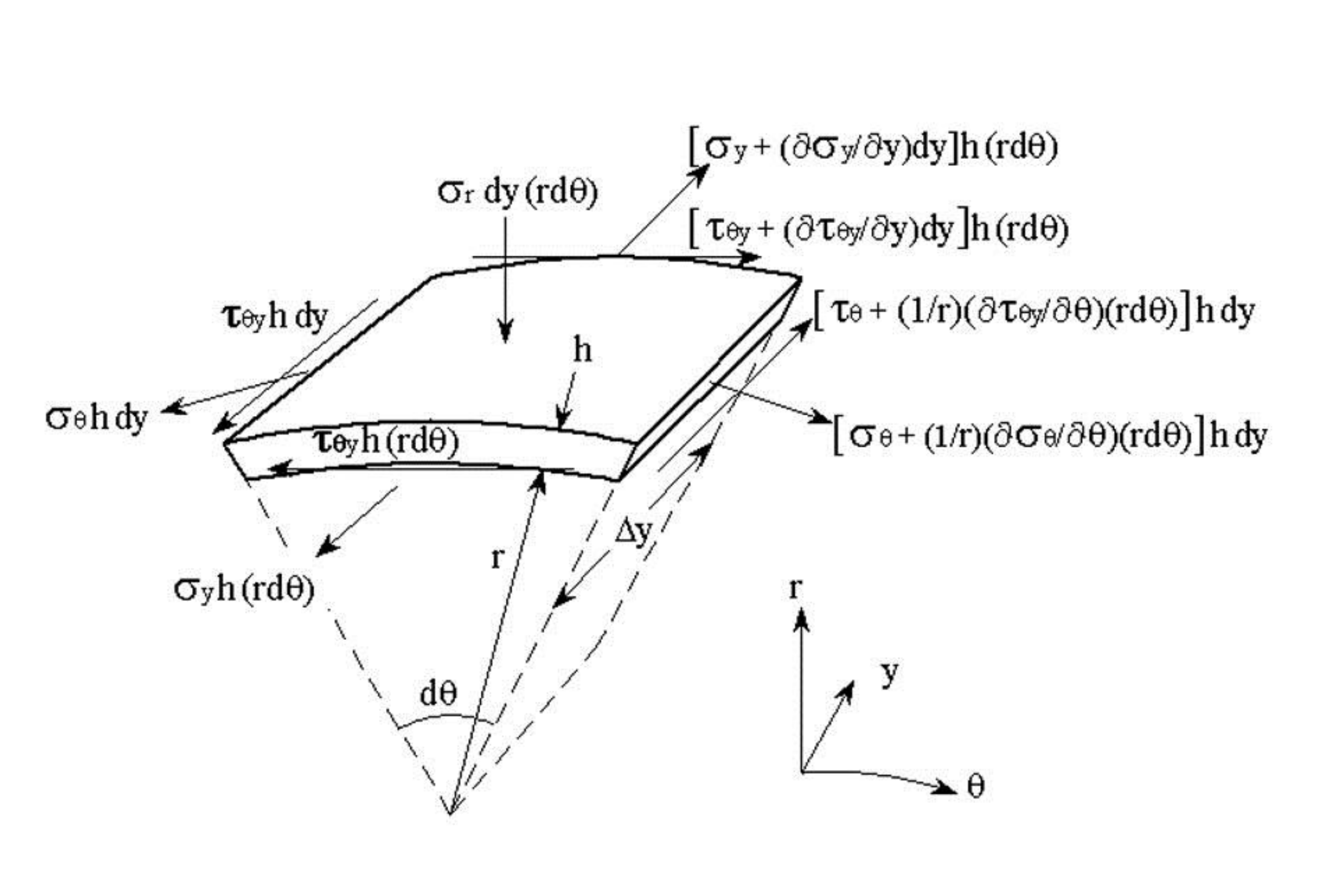
Generalized conditions for steady state flow of an elastic solid

- Furthermore, the strain in the direction of the particle trajectory must satisfy the requirement for constant mass flow. At every point in the web the following relationship between strain and velocity in the direction of the trajectory must be,

$$\frac{1+\varepsilon_{\psi}}{1+\varepsilon_{o}} = \frac{V_{\psi}}{V_{o}}$$

- If the nonlinear equations of elasticity are used, the strain subscripted with ψ is easily obtained as the strain corresponding to the deformed x coordinate.

Equations of equilibrium on a roller surface



Equations of equilibrium on a roller surface

- Equating forces in the θ direction.

$$\left(\frac{1}{r}\frac{\partial\sigma_r}{\partial\theta} r\,d\theta+\sigma_\theta\right)h\,dy-\sigma_\theta\,h\,dy+\left(\frac{\partial\tau_{\theta y}}{\partial y}dy+\tau_{\theta y}\right)h\,r\,d\theta-\tau_{\theta y}h\,r\,d\theta=F_\theta$$

- Equating forces in the y direction.

$$\left(\frac{\partial\sigma_y}{\partial y}dy+\sigma_y\right)h\,r\,d\theta-\sigma_y\,h\,r\,d\theta+\left(\frac{1}{r}\frac{\partial\tau_{\theta y}}{\partial\theta}r\,d\theta+\tau_{\theta y}\right)h\,r\,dy-\tau_{\theta y}h\,dy=F_y$$

- Since r is constant, $rd\theta$ may be replace by a single variable, which may as well be called x, the circumferential position.

Equations of equilibrium unwrapped

- Equations of equilibrium.
- In the x direction $$\left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) h = \frac{F_x}{dx\,dy} = S_x$$
- In the y direction $$\left(\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\right) h = \frac{F_y}{dx\,dy} = S_y$$
- The right hand terms may be thought of as friction stresses. They may be treated vectorially, like body forces.

The friction stress

- In the unwrapped model, the roller surface may be thought of as a flat plane under the web and with a normal component of pressure that was caused by the cylindrical geometry.
- The normal component of stress is a function of the MD stress.

$$\sigma_r (r\, d\theta)\, dy = \sigma_x\, h\, dy\, 2 \sin(d\theta / 2)$$

- So,

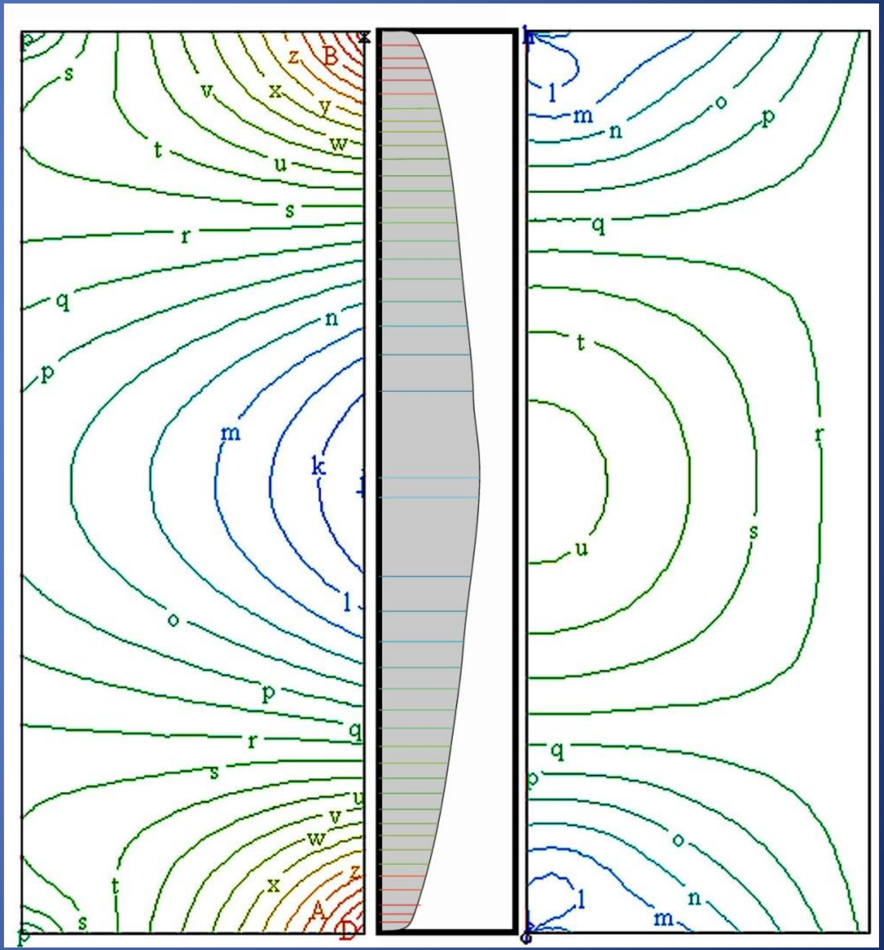
$$\sigma_r = \sigma_x \frac{h}{r}$$

The stick zone

- Except for the trivial case of a uniform web at a uniform, aligned roller, the surface friction of the roller must provide the reaction stresses required to meet the demands of the normal entry and normal strain rules.
- For example, in the case of a misaligned roller, there are shear stresses that must be balanced by something external to the web.
- In the case of a concave roller there are both shear and CD stresses to balance.

The stick zone

- Here's the concave roller example discussed earlier.
- The shaded area has been drawn in by hand to represent a hypothetical stick zone.
- The line of demarcation between the stick and microslip zones is a wild guess. All we can say with confidence is that there must be one.
- However, we can say something about the stress pattern in the stick zone.



The stick zone

- There is no x variation in the stress!
- You can imagine the web entering onto the roller as series of identical narrow strips parallel to the roller axis.
- So in the equations of equilibrium, the derivatives with respect to x will disappear and those with respect to y will be fixed.

$$\left(\frac{\partial \sigma_x}{\partial x} + \left. \frac{\partial \tau_{xy}}{\partial y} \right|_{entry} \right) h = S_x$$

$$\left(\frac{\partial \tau_{xy}}{\partial x} + \left. \frac{\partial \sigma_y}{\partial y} \right|_{entry} \right) h = S_y$$

The stick zone

- The values for the friction stresses, S_x and S_y, are now fixed as,

$$S_x = h \left. \frac{\partial \tau_{xy}}{\partial y} \right|_{entry} \qquad S_y = h \left. \frac{\partial \sigma_y}{\partial y} \right|_{entry}$$

- And for the web to stay in place on the roller, the vector sum of S_x and S_y must not exceed the maximum force per unit area created by friction. Therefore,

$$\sqrt{S_x^{\,2} + S_y^{\,2}} \le \frac{\varphi_s \sigma_x}{r} h$$

The stick zone

- The criterion for no slipping is therefore,

$$\sqrt{\left.\frac{\partial \tau_{xy}}{\partial y}\right|_{entry}^{2} + \left.\frac{\partial \sigma_{y}}{\partial y}\right|_{entry}^{2}} \leq \frac{\varphi_s \sigma_x}{r}$$

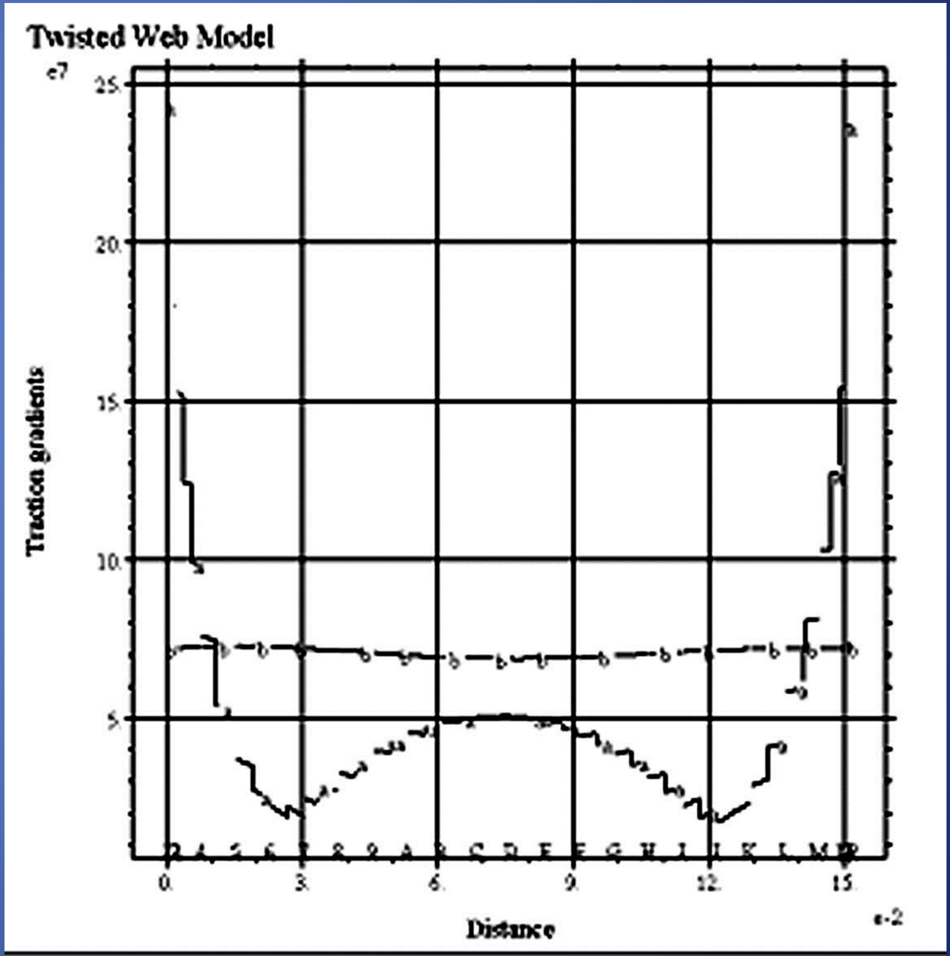
- This is only a *criterion* for *existence* of the stick zone. It says nothing about the direction or consequences of slipping.
- The term on the left will be called the stress rate and the one on the right the friction rate. Both have units of stress per unit length.

Application of the stick criterion to a twisted web experiment

- The next slide will show the result of applying the slip criterion to a twisted web experiment reported by Good and Straughan in 1999. They observed that at low tensions it took much more twist to create a wrinkle than at higher tensions – a situation similar to regime II wrinkling on a misaligned roller.
- The authors suggested that low tension led to low traction and this allowed the web to flatten on the roller.
- A typical low tension case was:
  - Twist angle = 5 degrees, span length = 0.108 m, width = 0.152 m, thickness = 23.4e-6 m, modulus = 4.13e9 Pa, tension = 26.7 N, coefficient of friction = 0.3, roller diameter = 0.0736 m

Application of the stick criterion to a twisted web experiment

- The curve labeled (b) is the friction rate.
- The curve labeled (a) is the stress rate.
- Wherever the stress rate exceeds the friction rate, slipping will occur.
- It's obviously slipping at the edges and very close to slipping everywhere else.
- All the low tension cases looked like this.
- In all the cases where "normal" behavior was observed, there was ample separation between the curves.

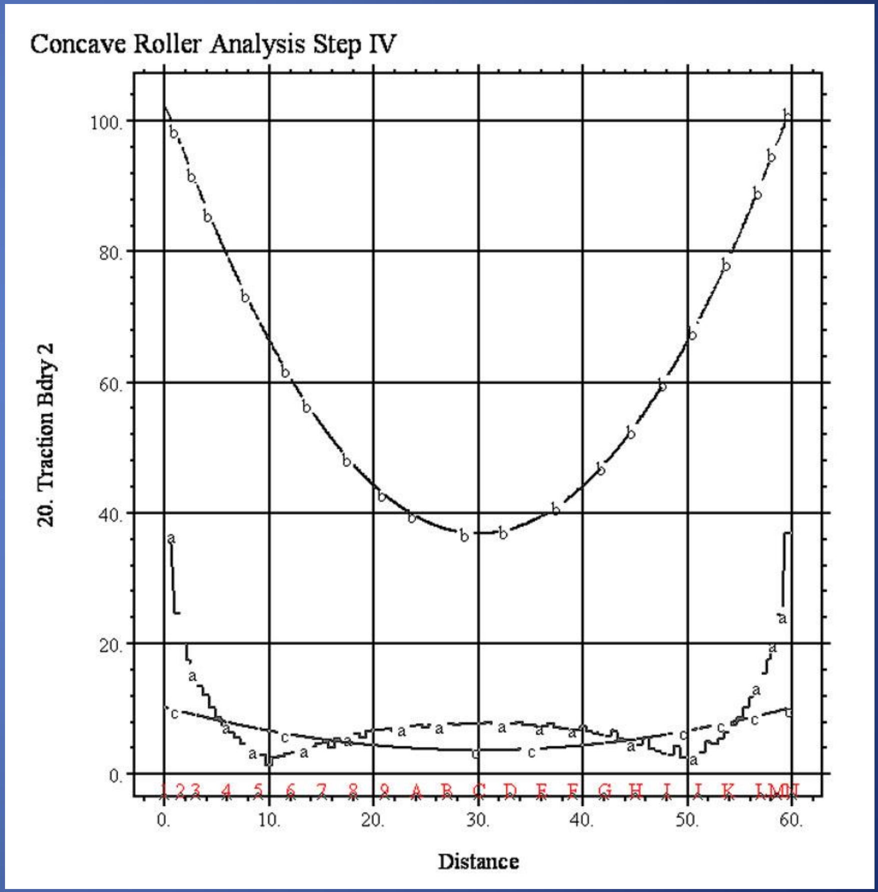


Application of the stick criterion to a concave spreader

- The next three slides will show how the criterion would work when applied to a concave roller. This is a hypothetical example which has not been confirmed by experiment.
- Application parameters are:
  - Span length = 20 inches
  - Width = 60 inches
  - Modulus = 50,000 psi
  - Poisson ratio = 0.35
  - Thickness = 0.001 inch
  - Roller diameter = 6 inches
  - The roller is 72 inches wide with a circular depth profile
  - Depth at the center = 0.05 inch
  - Results at three tensions are shown, 0.5, 1 and 2 pli

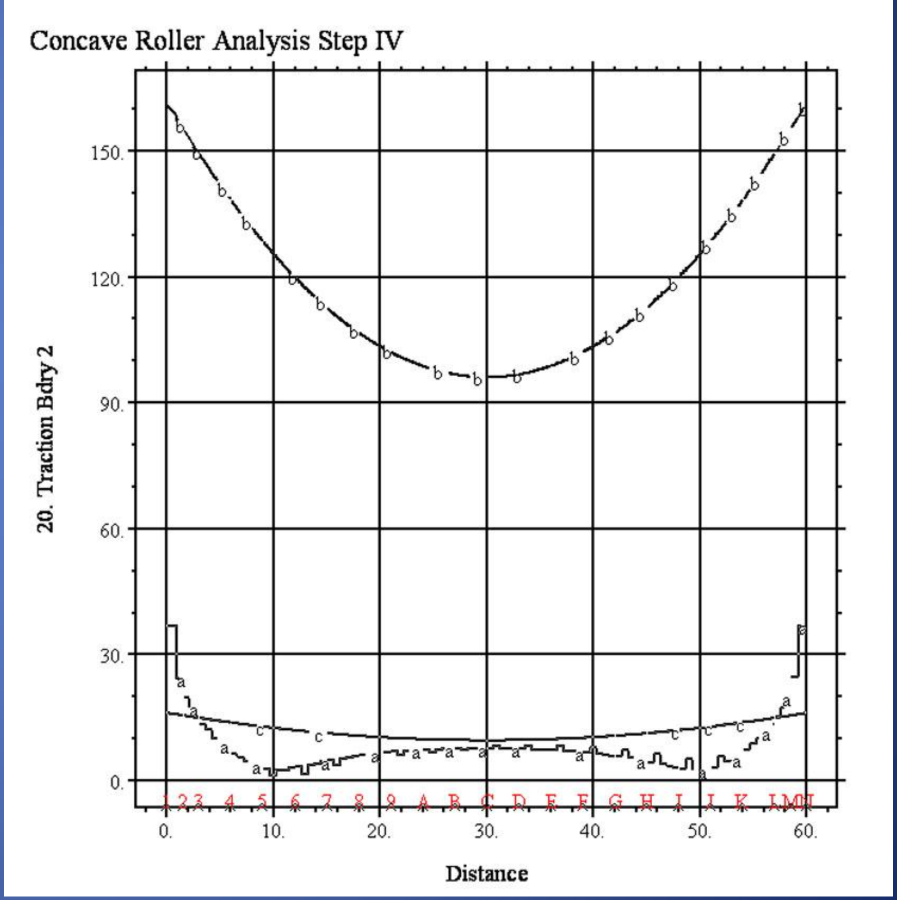
Application of the stick criterion to a concave spreader (0.5 pli)

- The curve labeled (a) is the stress rate.
- The top curve labeled (b) is the friction rate for a coefficient of friction of 0.35.
- The curve labeled (c) is the friction rate for a coefficient of 0.035.
- So at this tension, you could expect good results provided the line speed was low enough that air lubrication isn't a factor.



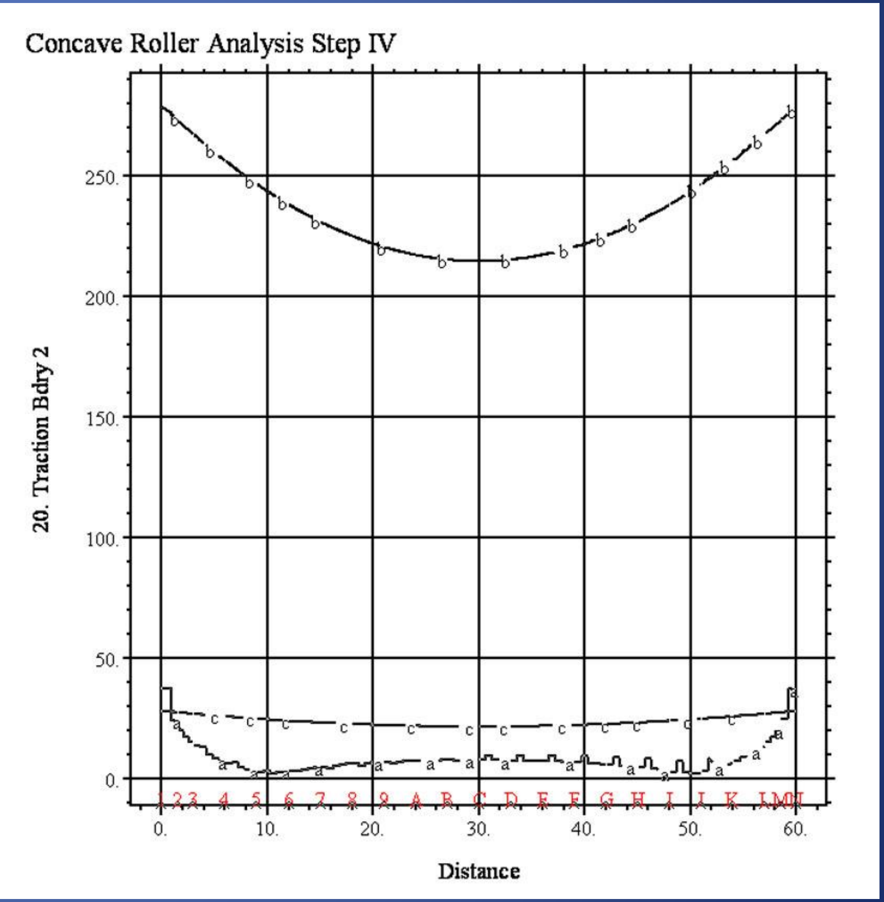
Application of the stick criterion to a concave spreader (1.0 pli)

- This is better. But, it's still marginal at a friction coefficient of 0.035



Application of the stick criterion to a concave spreader (2.0 pli)

- Increasing tension continues to improve the situation.
- An important point to note here is that the friction rate is directly proportional to the coefficient of friction and the MD stress. But, the *y-axis* stresses are usually influenced more by geometry than tension.



Application of the stick criterion to a cambered web

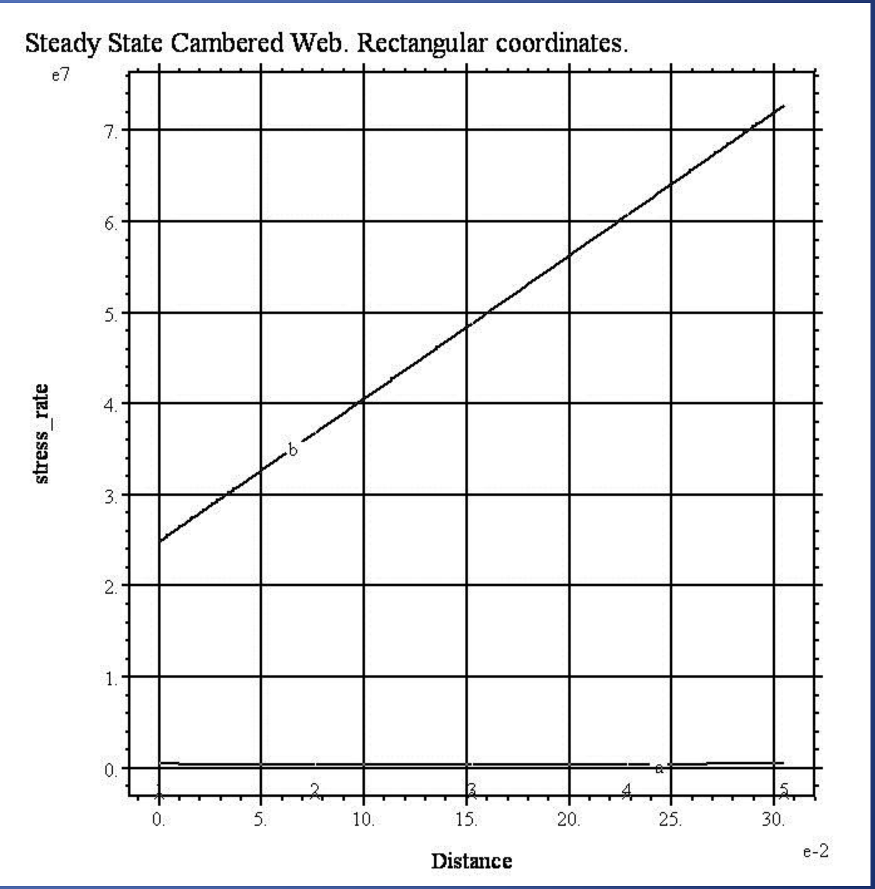
- Using the model presented in the author’s 2005 paper, “Effects of Concave Rollers, Curved-Axis Rollers and Web Camber on the Deformation and Translation of a Moving Web”, the stick criterion was applied to a cambered web.
- The application parameters were taken from a series of experiments reported by Swanson in his 1999 IWEB paper “Mechanics of Non-Uniform Webs”.

Application of the stick criterion to a cambered web

- The parameters for the first case are:
  - Span length = 2 m
  - Width = 0.305 m
  - Thickness = 23.4 microns
  - Radius of curvature of web = 139 m
  - Modulus = 3.45 GPa
  - Poisson ratio = 0.35 (assumed)
  - Avg tension = 66 N
  - Roller diameter = 7.6 cm
  - Coefficient of friction = 0.20

Application of the stick criterion to a cambered web

- The stress rate is insignificant compared to the friction rate. So, there is no chance of slipping.

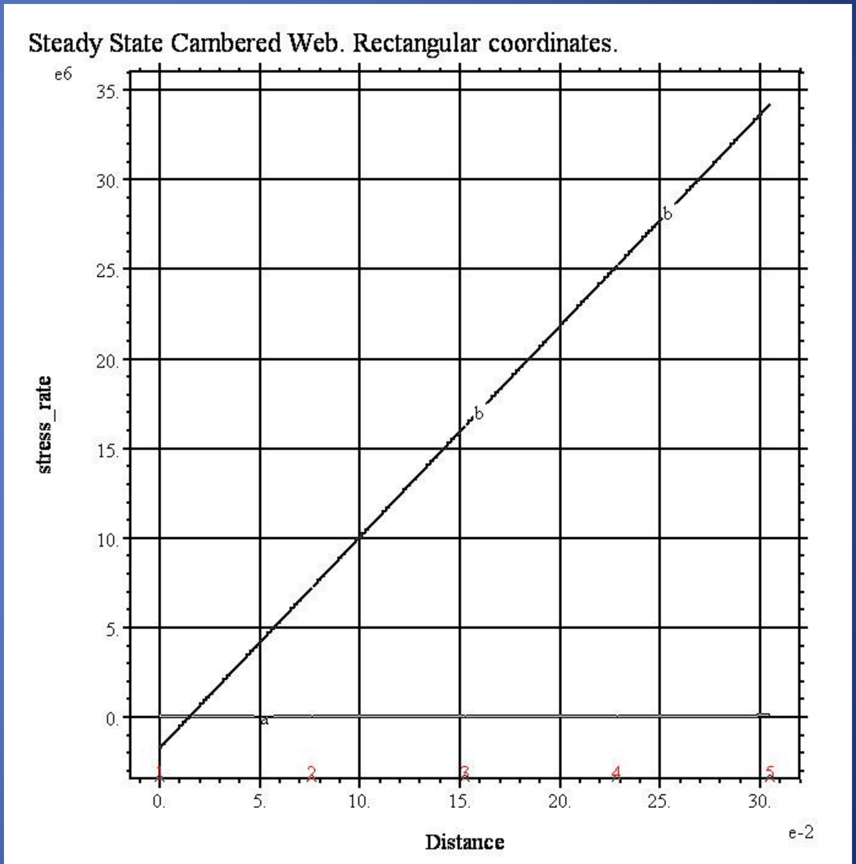


Application of the stick criterion to a cambered web

- Another test in the Swanson series is shown in the next slide. In this case the average tension was just high enough to avoid slackness on the long edge of the web.
- Application parameters were the same as the previous case except for the following,
  - Span length = 0.67 m
  - Radius of curvature of web = 185 m
  - Avg tension = 22 N

Application of the stick criterion to a cambered web

- As would be expected, the friction and stress rates cross at the long edge.
- This would obviously not be a good production situation.



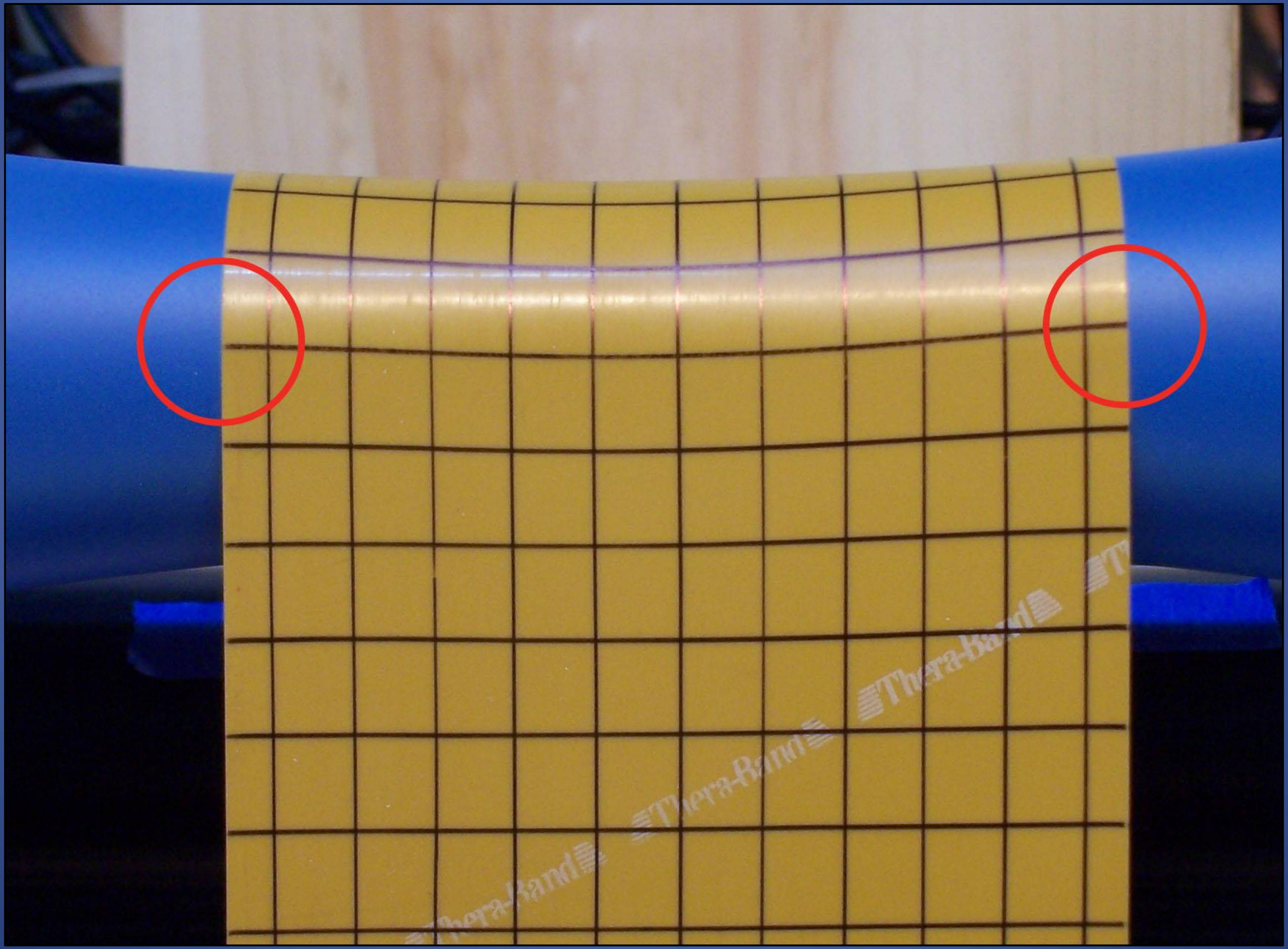
What happens if the web doesn’t satisfy the stick criterion?

- If the stick criterion can’t be met on some parts of the line of contact, then, at those places the normal entry and normal strain boundary conditions can’t be satisfied.
- Does the web just squirm around a bit at the entry and find a new stress state farther on that allows it to satisfy the boundary conditions?
- Consider the case of a concave roller.

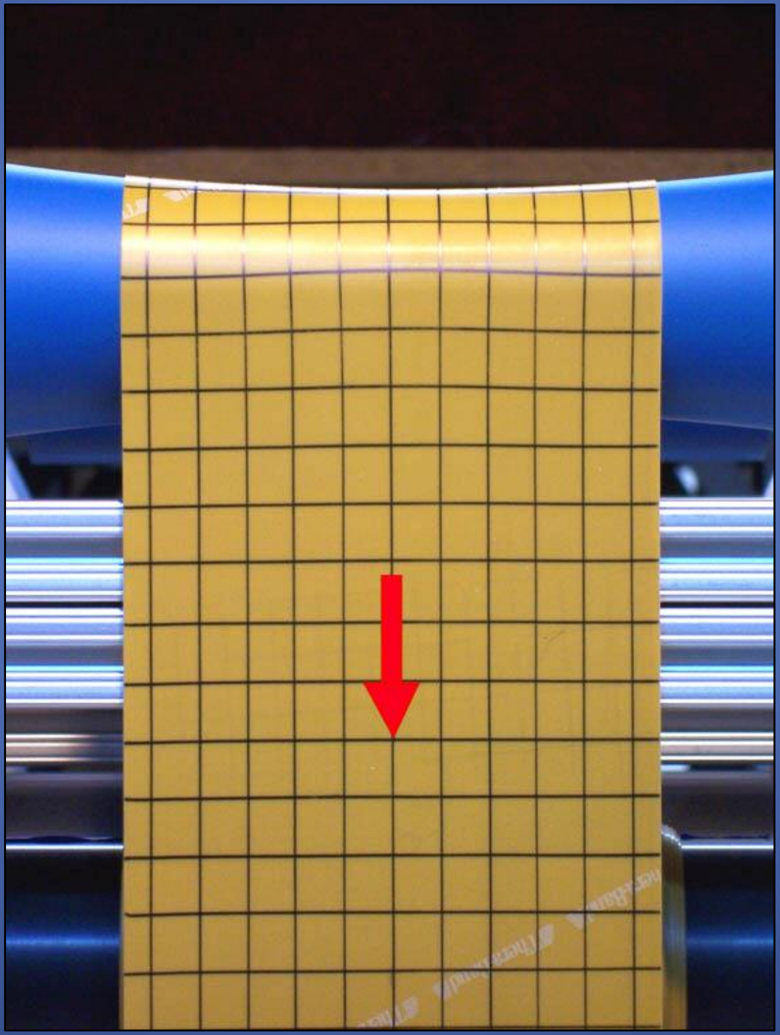
Slipping on a concave roller

- The reason a web spreads on a concave roller is that this is the only way it can deform so that it simultaneously satisfies the normal entry and normal strain conditions.
- The normal entry rule insures that in the steady state there will be no further lateral motion without slipping and the normal strain rule ensures that the mass flow rate at every point across the web is constant. If these conditions can’t be satisfied at the line of entry, how will they be satisfied farther in?

What happens if the web doesn't satisfy the stick criterion on a concave roller?



What happens if the web doesn't satisfy the stick criterion on a concave roller?



The microslip zone

- In the steady state, if the stick criterion can’t be met on some portion of the roller surface, then, the web must be moving relative to the surface at that location.
- Maybe it eventually adjusts its stresses so it can stick.
- But, in any event, conservation of mass must prevail.

- And that takes us to the microslip zone.

The microslip zone

- In the microslip zone the web is by definition slipping relative to the roller surface and we can be sure that the force of friction will be oriented to oppose that motion.
- Furthermore, the friction rate we developed in the stick zone discussion can be applied; but it will always have its maximum value. So,

$$\sqrt{S_x^2 + S_y^2} = \frac{\varphi_d \left|\sigma_x\right|}{r}$$

- There is now the question of direction.

The microslip zone

- The direction must be the same as the particle motion discussed at the beginning. The angle, ψ, relative to the x-axis is,

$$\psi = \tan^{-1}\left(\frac{\partial v}{\partial x} \frac{1}{1+\varepsilon_{\psi}} \right)$$

- With this information we can now write the equations of equilibrium for the microslip zone as,

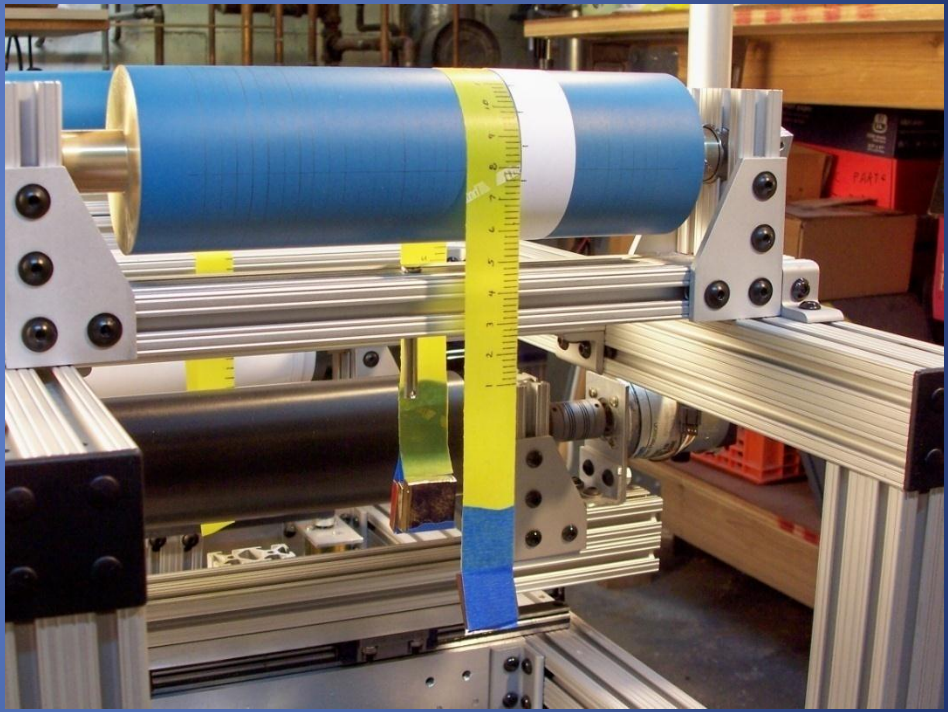
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \frac{\varphi_d \left|\sigma_x\right|}{r} \cos(\psi) \qquad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \frac{\varphi_d \left|\sigma_x\right|}{r} \sin(\psi)$$

The microslip zone

- Since the velocity vector of the particle motion must always be tangent to the particle paths (analogous to streamlines in a fluid), it seems reasonable to assume that ψ will always be small and the sign of the friction term will depend only on whether the roller is driving (web velocity less than the roller surface) or braking (web velocity greater than the roller surface).
- This seems wonderful at first. Equations of equilibrium have now been developed for both zones. Unfortunately, something more is needed.
- The choice of equilibrium equations, stick or microslip, depends on knowing where the transition from stick to slip occurs. But, the line of transition isn't known until the solution is available. The problem is illustrated in the following slide.

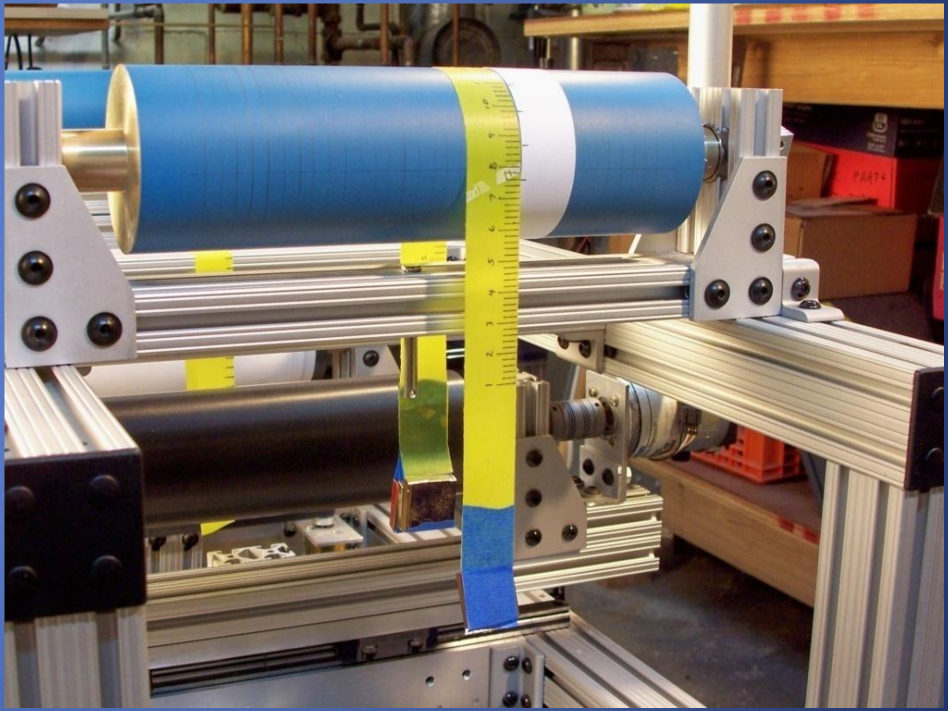
The microslip zone

- If a thin latex band is draped over a roller with weights on each end to produce tension, a microslip zone will develop on each side.
- If progressively more weight is added to one side, the microslip zone on that side lengthens until it intersects the other and the band then slips off the roller.



The microslip zone

- If instead of adding weight, the roller is rotated, the upstream microslip zone is consumed until only a single longer one exists on the downstream side.
- In both cases the total tension differential is supported by microslip.
- This behavior is a challenge to model with existing tools.

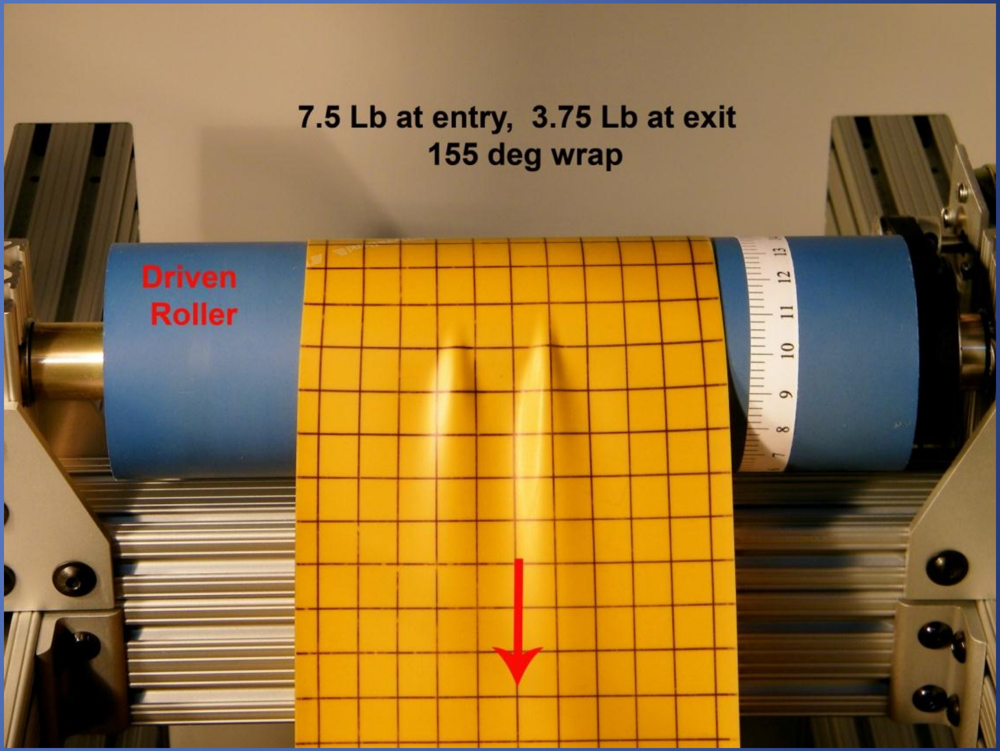


The importance of the microslip zone

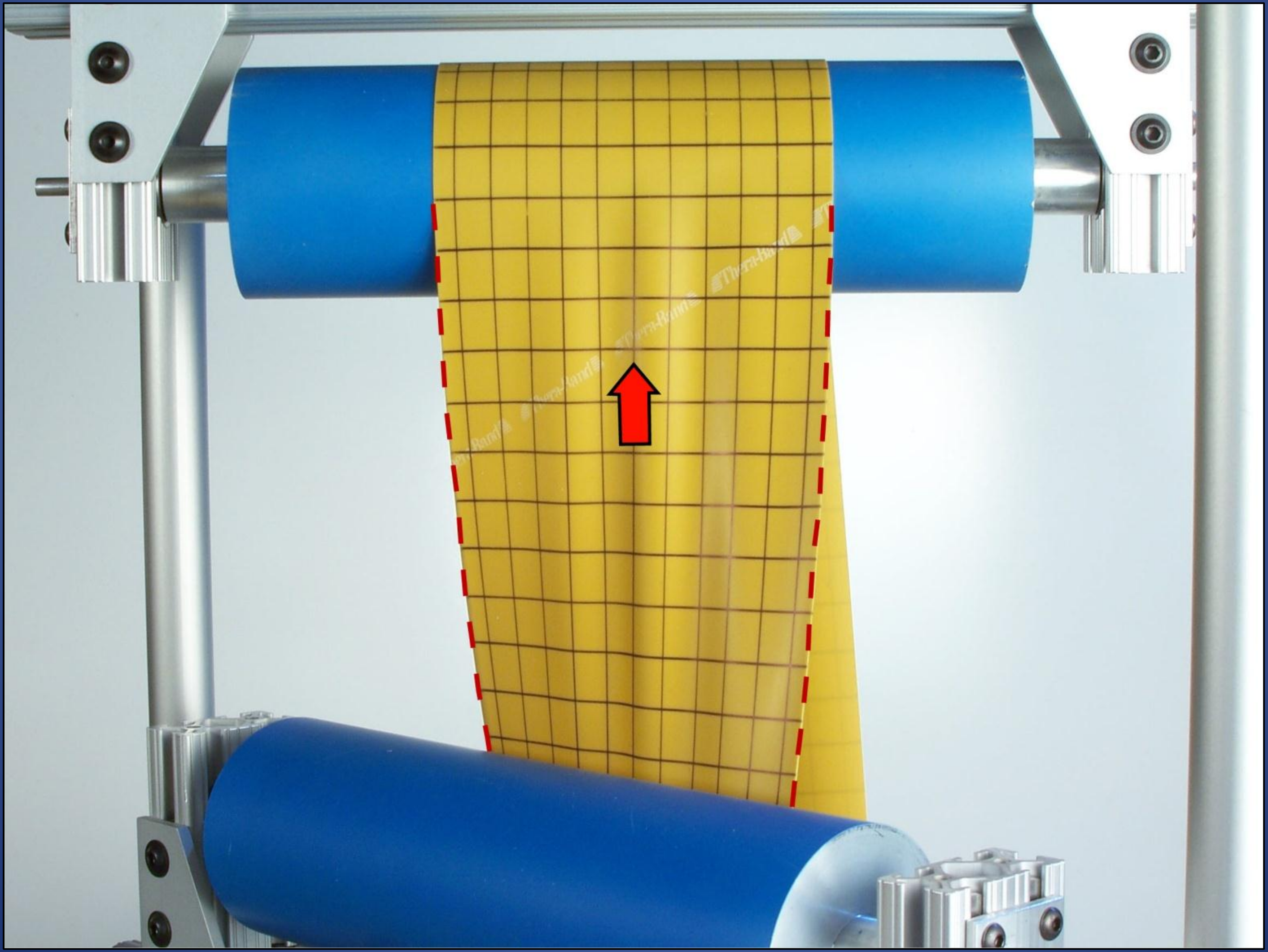
- It is the zone in which torque is transferred between the roller and the web.
- It is the zone where stresses transferred from the previous span have their first effect.
- Nonuniform stress downstream of a roller can cause part of a microslip zone to extend all the way to the line of entry, thus invalidating assumptions of isolation from downstream effects. Furthermore, the downstream effects could be caused, in part, by strain that is transferred from the upstream side.

Some interesting observations of web behavior on rollers

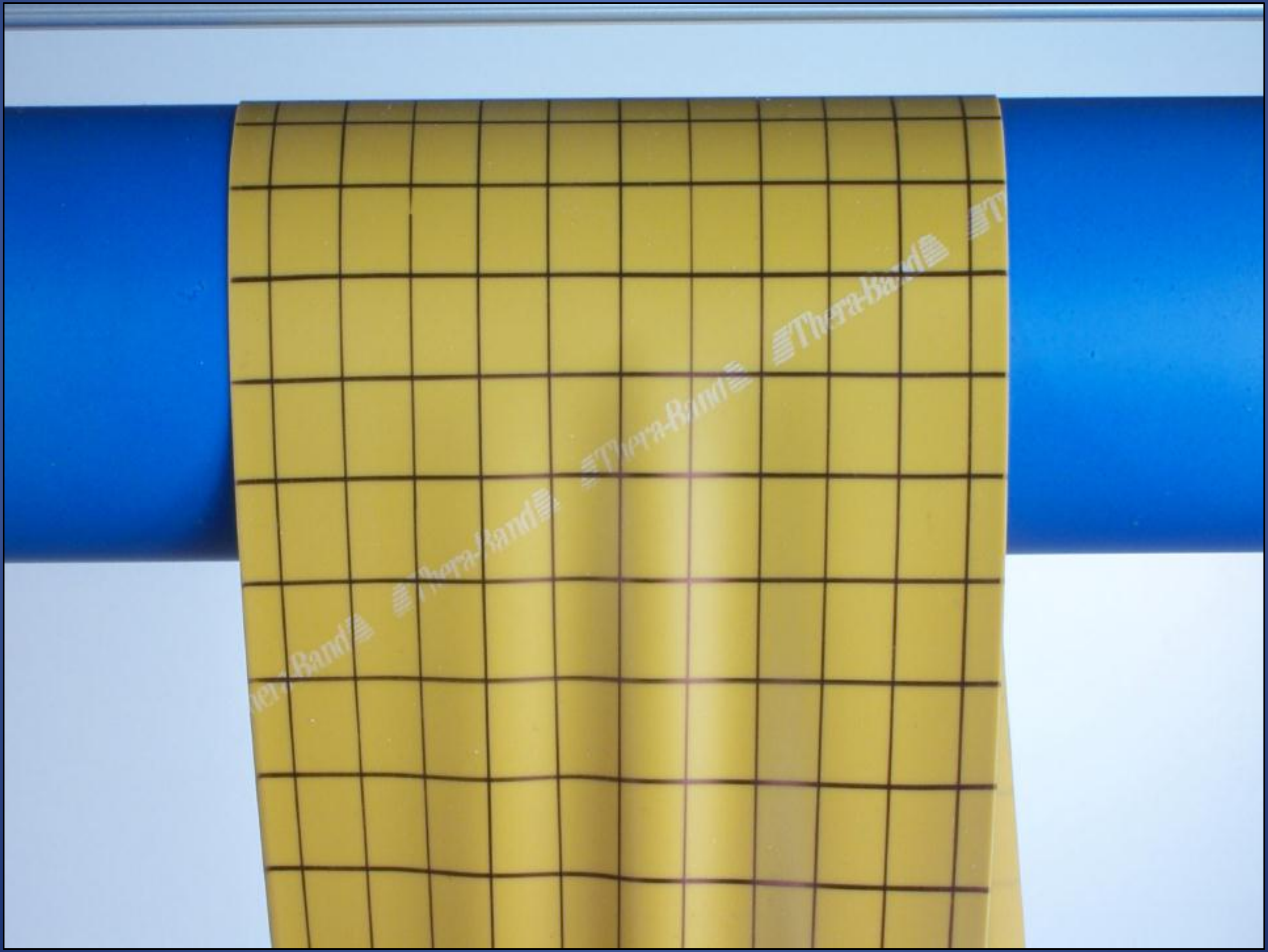
- Troughs at the exit of a driven roller.



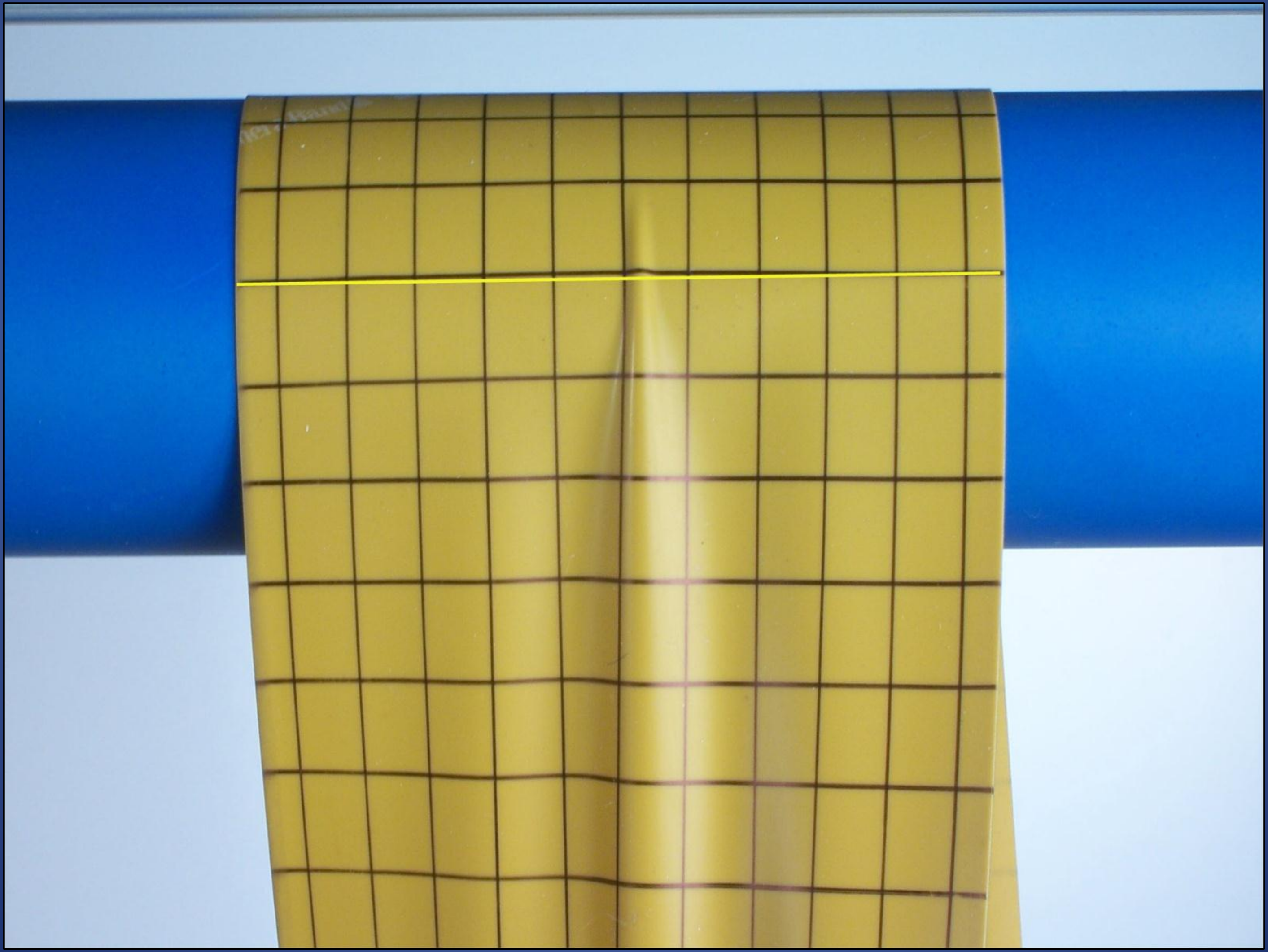
Wrinkle formation



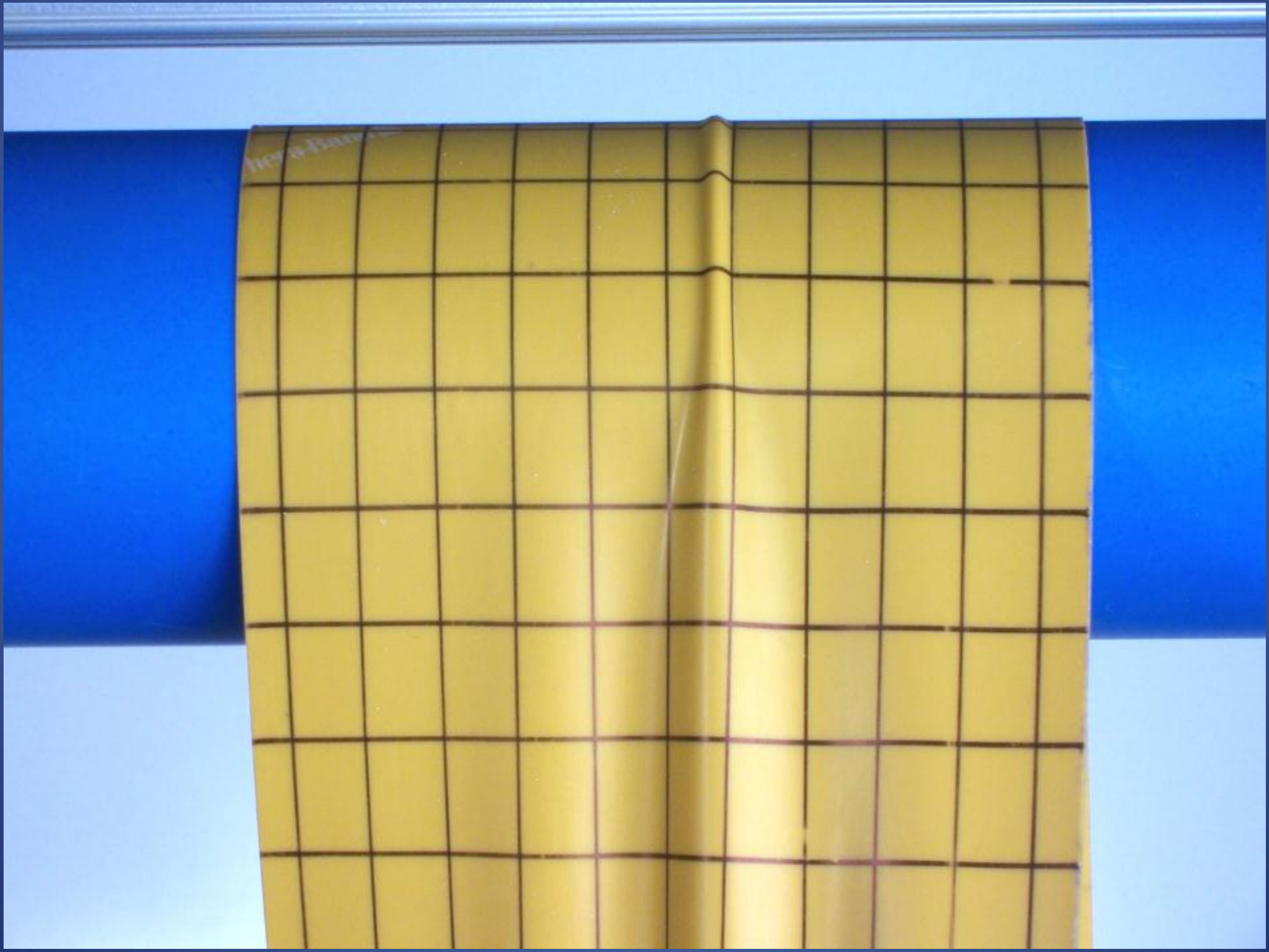
Wrinkle formation



Wrinkle forming



Greasing the roller



Conclusions

- A general model for two-dimensional steady state flow of an elastic web has been presented.
- Equations of equilibrium, including friction, for a web on a roller (straight but not necessarily uniform) have been developed from first principles.
- It has been mathematically demonstrated that if a web on a roller is flexible enough to be treated as a membrane and remains in contact with the roller, it may be treated as though it is flat. It should be noted that the ability to make this transformation suggests that when bending stiffness can be ignored, the cylindrical shape of the web on a roller imparts no special mechanical attributes to it.

Conclusions

- A two-dimensional criterion has been established for the existence of a stick zone at the entry to a roller. This criterion is, in effect, a mathematical definition of the stick zone.
- Applications of the stick criterion to concave rollers and a cambered web have been illustrated.
- Equations of equilibrium for the microslip zone have been developed. But, it is not yet clear how to incorporated them into a comprehensive model that can be numerically modeled.
- Photographs illustrating wrinkling at both the entry and exit of a roller have been presented.

Thank you

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